Exercises for
Financial Mathematics

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Important: Open the file in Adobe Reader in order to toggle on/off the answers.

Contents

Introduction

For each question write down which approach you are using, the formula required, the knowns and the unknowns, as well as your derivation and answers. An example would be as follows:

Question: If you place 100 for a duration of 11 months into a bank account at an annual interest rate of 10%, then what is the total amount in your bank account at the end of the 11 months?

Answer: Your answer should always follow this approach. Very often it is useful to draw a graph in order to see that you fully understand the different time schedules involved. I strongly advise you to do that.

1. Approach: simple interest (less than one year)
2. Formula: $(1 + rt)P = S$
3. Knows: $P = 100$, $t = 11/12$, $r = 0.1$
4. Unknowns: S
5. Then you substitute everything in the formula: $(1 + 0.1\frac{11}{12})100 = S$. You solve: $S = 109.17$ (you round to the next cent).
6. You write down the solution in a full sentence: If you place 100 for a duration of 11 months into a bank account at an annual interest rate of 10%, then the total amount in your bank account at the end of the 11 months is 109.17.

Some further points/reminders:

- I am using the UK/US accounting way to write numbers, meaning a 'dot' is used for the decimal separator, while a 'comma' is used as a three-digit separator. Thus, the value 1000000 and 25 cents is written as 1,000,000.25.
- Days in a year according to convention (ordinary interest): 360
- Be careful about the starting and the end dates, e.g. there is a difference between 'beginning' and 'end' of year. Example: from beginning of 2019 to beginning of 2020 corresponds to one year; while from beginning of 2019 to end of 2020 corresponds to two years.
- Only round the final solution to the nearest cent (if necessary), do not round before. For x.xxy, round up if $y \geq 5$, round down if $y < 5$. Example: 1,003.045 becomes 1,003.05, while 1,003.043 becomes 1,003.04.

1 Exponents and equation solving

1. Use all four laws of exponents to transform each of the following (there are many possibilities)

   1.1. Example: 16. We can re-write this as $2^4 = 2^2 2^2 = 2^4 2^{-2} 2^2 = 2^6/2^2 = (2^2)^2 = (2 \times 2)^2$. And thus we applied (randomly) the four laws of exponents.

   1.2. 8^4.
   Answer: $8^2 8^2 = 8^2/8^{-2} = 8^{2+2} = (2^2 \cdot 2)^{2+2} = (2^2)^4 \cdot 2^{2+2}$.

   1.3. $(xy)^{-2/3}$.
   Answer: $(xy)^{-2/3} = x^{-2/3} y^{-2/3} = \frac{1}{(xy)^{2/3}} = ((xy)^{-2})^{1/3}$.

   1.4. 10^{-10}.
   Answer: $10^{-10} = \frac{1}{10^{10}} = (2 \cdot 5)^{-10} = 2^{-10} \cdot 5^{-10} = (10^{-2})^5 = (10^{-2})^5 = \left(\frac{1}{(100)^{1/2}}\right)^{10}$.

   1.5. $(x^x)^{2x}$
   Answer: $(x^x)^{2x} = x^{2x^2} = \left(\frac{1}{x^{-x}}\right)^{2x} = \left(x^{x^2}\right)^2$.

2. Solve for P.

   2.1. $(1 + 0.3 \cdot 5)P = 15500$.
   Answer: $P = 6200$.

   2.2. $(1 + 0.3)^5 P = 15500$.
   Answer: $P = 4174.6$.

   2.3. $(1 + 0.01)^{3/4} P = 2$.
   Answer: $P = 1.98513$.

   2.4. $(1 + 0.01 \cdot 3/4)P = 2$.
   Answer: $P = 1.98511$.

   2.5. $(1 + 0.025 \cdot 2.9)P = 18.9$.
   Answer: $P = 17.6224$.

   2.6. $(1 + 0.025)^{2.9} P = 18.9$.
   Answer: $P = 17.5939$.

3. Solve for t.

   3.1. $10^t = 2$.
   Answer: $t = \log_{10} 2$, thus $t = 0.30103$.

   3.2. $(1 + 0.3)^t = 4.5$.
   Answer: $t = \log_{1.3} 4.5$, thus $t = 5.73278$.

   3.3. $2 = e^t$.
   Answer: $t = \ln 2$, thus $t = 0.693147$.

   3.4. $3 = e^{t+2} - 3$.
   Answer: $t = \ln(6/e^2)$, thus $t = -0.208241$.

4. Solve for r.

   4.1. $(1 + r5)10000 = 15500$.
   Answer: $r = 0.11$.

   4.2. $(1 + r)^5 10000 = 15500$.
   Answer: $r = 0.0916071$.

   4.3. $(1 + r)^{3/4} 1.5 = 2$.
   Answer: $r = 0.467523$.

   4.4. $(1 + r3/4)1.5 = 2$.
   Answer: $r = 0.444444$.

   4.5. $(1 + r2.9)18 = 18.9$.
   Answer: $r = 0.0172414$.

   4.6. $(1 + r)^{2.9} 18 = 18.9$.
   Answer: $r = 0.0169665$.

5. Solve the following equations for the unknown.

   5.1. $(1 + x)^2 = 1600$.
   Answer: $x = -41$ or $x = 39$.

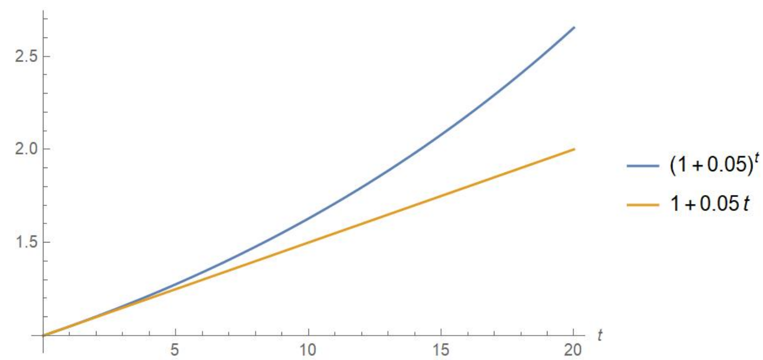
   5.2. $2(1 + x)^{1/2} + 5 = 11$.
   Answer: $x = 29.25$.

5.3. $2\Big((1+x)^{1/2}+5\Big) = 16.$
Answer: $x = 8.$

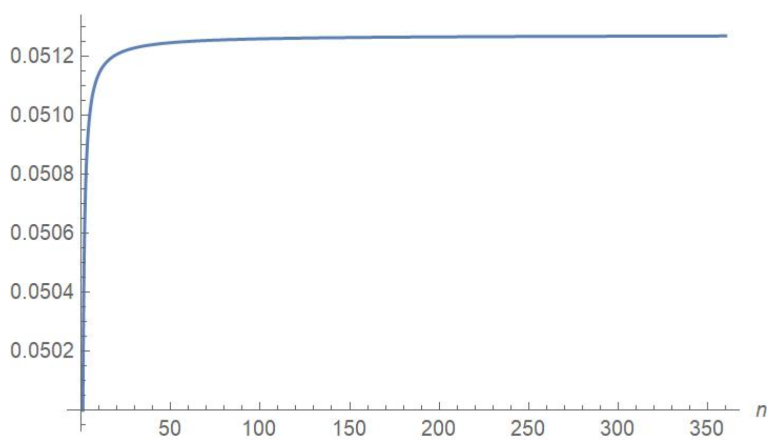
6. Graph the following equations.

6.1. $y_t = (1+0.05t)$, for $t \in [0, 20]$.

6.2. $y_t = (1+0.05)^t$, for $t \in [0, 20]$.
Answer: Here we graph the answers to the two equations together.



6.3. $y = (1+0.05/n)^n - 1$, if $n \in [1, 360]$. Also plot the value of y_∞.
Answer: Here is the graph corresponding to this equation. You can notice how it starts at 0.05 for $n = 1$, and it converges to 0.0512711 for $n = \infty$.



2 Simple (short term) interest rates

For each question write down the formula used, the knowns and the unknowns, as well as your derivation and answers.

1. What is the interest on a 20 day loan of 1,000€

   1.1. at an interest rate of 5%?
   Answer: $I = 2.78$€.

   1.2. at an interest rate of $r = 0.01$?
   Answer: $I = 0.56$€.

2. How long will it take for 1,000€ to accumulate to 2,500€ if

   2.1. the simple interest rate is 5%?
   Answer: 30 years

   2.2. the simple interest rate is $r = 0.015$?
   Answer: 100 years

3. Jacky borrowed money. One year after borrowing this money he pays back 500€. How much did he borrow if

   3.1. the interest rate is 15%?
   Answer: $P = 434.78$€

   3.2. the interest rate is $r = 0.015$?
   Answer: $P = 492.61$€

4. Sophie borrows 500,000€ for 83 days. How much does she have to pay back and what is the interest if

   4.1. the interest rate is 3%?
   Answer: $S = 503,458$€, $I = 3,458$€

   4.2. the interest rate is $r = 0.012$?
   Answer: $S = 501,383$€, $I = 1,383$€

5. What is the formula for calculating the accumulated value based on simple interest rate? Write it from your memory ten times. And add the formula for calculating the interest.

6. Simon borrows 3,000€ from Yann at an interest rate of 5%. He needs to repay this money in 6 months from now. Assume Yann can then invest that money at a rate of 10%. How much would this be worth

   6.1. 12 months from now?
   Answer: $3,228.75$€

   6.2. 7 months from now?
   Answer: $3,100.63$€

   6.3. 3 months from now?
   Answer: $3,000$€

7. Eric needs to give 3,000€ to Agustin in 1 year and 500€ to Madou in six months. Assume the interest rate is 5%. How much does Eric need to give them if he wants to give them the money

   7.1. today?
   Answer: 2857.14€ and 487.81€

   7.2. in 1 months?
   Answer: 2868.53€ and 489.8€

   7.3. in 12 months?
   Answer: $3,150$€ and 525€.

   (So the idea is that both Agustin and Madou are indifferent between getting the money later and at another date.)

8. Yann makes a contract with Marie: He borrows 1,000€ from Marie for 120 days at an interest rate of 100%. Immediately after making the contract Marie's car breaks down and she needs some money urgently. She thus wants to sell the contract to Simon. What would be the maximum amount of money that Simon would be willing to give to Marie if he could instead invest his own money

8.1. at 15% interest rate during the same period?
Answer: $1,269.84$€

8.2. at 5% interest rate during the same period?
Answer: $1,311.47$€.

8.3. Explain the result.

Answer: So the idea is the following. If Simon were to invest $1,269.84$€ now at an interest rate of 15%, then he would get the $1,333.33$€ which is the same amount that he would get if he were to buy the contract from Marie. So he would definitely not give more money to Marie than $1,269.84$€ because then he could simply invest this money at the 15% interest rate and be better of.

9. What is the interest rate if an investment of 115,000€ accumulates to 120,000€ in 9 months?
Answer: $r = 0.058$€

10. You place 1,200€ into a bank account on the 1st of April. On which day does your bank account hold 1,250€ if the interest rate is 15%?
Answer: A hundred days later, which is 10 July.

11. You place 1,200€ into a bank account for 30 days at an interest rate of 6%. After the 30 days the interest rate changes to 4%. You then add 400€ and leave everything in the bank for another 40 days. What is the final sum of money in your bank account?
Answer: $1,613.14$€

12. Two investments are undertaken with a combined sum of 10,000€ at an interest rate of 3%. One of the investments lasts for 30 days, the other for 120 days. The accumulated value of the investment lasting 120 days is 10 larger than the accumulated value of the other investment. What is the total interest of the two investments?
Answer: I solve this one fully for you. Two investments, same interest rate, thus we have $S_1 = (1 + rt_1)P_1$ and $S_2 = (1 + rt_2)P_2$. We know that $P_1 + P_2 = 10,000$, $t_1 = 30/360$ and $t_2 = 120/360$, and $r = 0.03$. Then $10S_1 = S_2$. We

need to find the total interest, so that would be $I_1 + I_2 = I$. So we solve this as a system of equations, substituting one after the other, and we get $P1 = 915.27$€, $S1 = 917.56$€, $P2 = 9,084.73$€, $S2 = 9,175.58$€. Then the total interest is $rt_1P_1 + rt_2P_2 = 0.03(30/360)915.27 + 0.03(120/360)9,084.73 = 93.14$€.

13. A Mrs. Debtor writes a promissory note to a Mr. Creditor: "In 80 days from today I, Mrs. Debtor, promise to give Mr. Creditor 20,000€ at an interest rate of 5% per annum." What sum will Mr. Creditor receive?
Answer: $S = 20,222.2$€

14. An investor owns a bill of exchange stating a value of 50,000€ due on the 20th October. On the 20th June he needs money and sells the bill of exchange to a bank that works with an annual interest rate of 3%. Which amount of money does the investor get from the bank?
Answer: $49,505$€

15. A promissory note states a value of 20,000€ to be paid on the 18th December. It is sold at 19,431.6€ on the 2nd of January of the same year. What was the interest rate used.
Answer: $r = 0.0304$.

16. An investor has two bills of exchange, the first promises to pay 200,000€ on the 20th of June, the second one promises to pay 180,000 on the 12th of July. On the 2nd of February he wants to exchange these two bills of exchange for a single one that promises to pay a certain amount of money on the 4th December. The interest rate used is 8%. What will be the amount of money that he promises to pay on the 4th December?
Answer: $392,557$€

17. On the 1st January, in order to simplify the accounting, a company wants to replace two bills of exchange, one over 2,500€ due on the 31st March, the other over 3,000€ due on the 30th of April, with one of value 5,500€ calculated at

an interest rate of 6%. Determine the maturity of this new bill of exchange.
Answer: $t = 105.327$, which implies 16th of April.

3 Compound (long-term) interest rates

1. How much capital is in the bank account at the end of 2019 if a client deposited 120,000€ at a 4% interest rate compounded annually at the beginning of 2019?
   Answer: $S = 124,800$.

2. How much capital is in the bank account at the beginning of 2025 if a client deposited 120,000€ at a 4% interest rate compounded annually at the beginning of 2019?
   Answer: $S = 151,838$.

3. How much capital is in the bank account at the end of 2025 if a client deposited 120,000€ at a 4% interest rate compounded annually at the beginning of 2019?
   Answer: $S = 157,912$.

4. How much capital is in the bank account at the beginning of 2025 if a client deposited 120,000€ at a 4% interest rate compounded annually at the end of 2019?
   Answer: $S = 145,998$.

5. How much capital is in the bank account in 2025 if a client deposited 120,000€ at a 4% interest rate compounded annually in 2019?
   Answer: $S = 151,838$.

6. How much capital is in the bank account in 2025 if a client deposited 120,000€ at a 4% interest rate compounded quarterly in 2019?
   Answer: $S = 152,368$.

7. How much capital is in the bank account in 2025 if a client deposited 120,000€ at a 4% interest rate compounded continuously in 2019?
Answer: $S = 152,550$.

8. What is the annually compounded interest rate if a bank lends 20,000€ over two years and the borrower needs to repay 22,500?
Answer: $r = 0.06066$.

9. In the year 52 B.C. the Roman emporer Julius Caeser lent two gold coins to one of your ancestors. He asked for an interest rate of 1%. Last month a descendant from Caeser finds the stone on which this was engraved and gives you until 2020 to repay the debt, otherwise he'll send his friend Brutus around to talk to you. Assume the exchange rate between gold coins and was constant at 1 gold coin equal to 2€. How much would you need to repay the descendant? Solve this once assuming interest is compounded annually and once for simple interest.
Answer: Simple interest: $S = 86.92$.
Compound interest: $S = 3,633,020,000$.

10. How long does it take to accumulate 100,000€ if a client deposited 80,000€ at a 4% interest rate compounded quarterly?
Answer: You solve $100,000 = 80,000(1 + 0.04/4)^{4t}$. First simplify: $1.25 = (1+0.04/4)^{4t}$. Use logarithms on both sides with the Potency Rule: $\log(1.25) = 4t \log(1 + 0.04/4)$. Then isolate t: $t = \frac{\log(1.25)}{4\log(1+0.04/4)}$. This gives $t = 5.60644$. That's five years and 0.606 years. This is $0.606 = \frac{0.606\times360}{360} = \frac{218.16}{360}$. So it's five years and 218 days.

11. Which of the following investments is more attractive?

    11.1. 6% compounded monthly

    11.2. 6.1% compounded quarterly

    11.3. They are the same.

    Answer: We assume the principal and time is the same across the investments and then compare the accumulated value. $S = (1 + \frac{0.06}{12})^{12}$ gives $S = 1.06168$, while $S = (1 + \frac{0.061}{4})^4$ gives $S = 1.06241$.

12. How much does Danielle have at the end of 2 years if she invests 1,200€ at 5.5% interest rate compounded annually? How much if interest is compounded continuously?
Answer: With the annually compound interest she gets $S = 1335.63$ and with the continuously compound she gets $S = 1339.53$.

13. At the beginning of 2010 a client placed 30,000€ in a savings account at an annually compounded interest rate of 2%. Since then he withdrew 20,000€ at the beginning of 2013, he withdrew 5,000€ at the end of 2013, and he deposited 15,000€ at the beginning of 2015 and another 10,000€ at the end of 2018. How much money does he have in his savings account at the end of 2019?
Answer: We calculate: $S = \Bigg(\Bigg(\Big((30,000(1+0.02)^3 - 20,000)(1+0.02) - 5,000\Big)(1+0.02) + 15,000\Bigg)(1+0.02)^4 + 10,000\Bigg)(1+0.02)$, which yields $S = 34,726.5$.

14. A principal of 200,000€ was placed in an account at the beginning of 2008 at an annually compounded interest rate of 4%. The interest rate changed to 2.5% at the end of 2012. What is the capital at the end of 2019?
Answer: We calculate $S = 200,000(1+0.04)^5(1+0.025)^7$, which is $S = 289,244$.

15. You have high hopes to create a family and buy a house when you are older. In order to do so you estimate that you need to have around 1,200,000€ to buy a house at the end of 2035. Assume your bank gives you an interest rate of 5%. How much money do you need to place into the bank account at the beginning of 2019 in order accumulate the desired amount by 2035?
Answer: $S = 1,200,000/(1+0.05)^{17}$, which gives $S = 523,556$.

16. Johnny invested 600€ at the rate of 8% interest compounded monthly. Find the value of his investment after 10 years. How much is the value if interest is compounded continuously?

Answer: $S = 600\left(1+\frac{0.08}{12}\right)^{12\times10}$, which is $S = 1,331.78$. If interest accumulates continuously, he will have $S = 1,335.32$.

17. Agustin invested 1,000€ at a rate of 9% interest compounded quarterly. Find the value of his investment after 5 years. How much is the value if interest is compounded continuously?
Answer: For the quarterly compounded case we find $S = 1,560.51$ and for the continuously compounded it is $S = 1,568.31$.

18. How much money do you need to invest if you want to have 10,000€ in 3 years at an interest rate of 6.5% compounded annually. How much is the value if interest is compounded continuously?
Answer: In the annually compounded case it is $P = 8,278.49$ and in the continuously compounded case it is $P = 8,228.35$.

19. An investor wants to do the following transfers at a monthly compounded interest rate of 4.5%:

   - January 2009: deposit 5,000€
   - January 2012: take out 4,000€
   - January 2014: deposit 5,000€

   What is the amount in the bank account at the end of 2018?
   Answer: $S = \left(\left(5000(1+\frac{0.045}{12})^{12\times3}-4000\right)(1+\frac{0.045}{12})^{12\times2}+5000\right)(1+\frac{0.045}{12})^{12\times4}$, which gives $S = 8,237.7$.

20. Find the amount accumulated after investing a principal of 1,520€ for 6 years at the annual interest rate of 5.4% compounded 12 times in a year. How much is the value if interest is compounded continuously? What is the interest in both cases?
Answer: $S = 1520(1+0.054/12)^{12\times6}$, which is $S = 2100.1$, with $I = 2100.1 - 1520$. Under continuous compounding we get $S = 2101.62$, with $I = 2101.62 - 1520$.

21. Joe has 1,200€ to invest at 6% annual interest compounded monthly. How long will it take for his investment to grow to 5,000€? How much is the value accumulated during the same time if interest is compounded continuously?
Answer: We find $t = 23.8447$, which is 23 years and $0.8447 \times 360 = 304$ days. The value accumulated from investing 1200 at 6% annual interest compounded continuously would be $S = 5,017.86$.

22. Danielle has 15,300€ to invest at a 3.5% semi-annual interest rate compounded daily. How long will it take for the investment to grow to 25,000€? How much is the value accumulated during the same time if interest is compounded continuously?
Answer: We find $t = 7.0153$, which is 7 years and $0.0153 \times 360 = 5.5$ days. At continuous compounding we'd have $S = 25,001.2$.

23. Yann is planning to invest 1,280€. Find the annual interest rate compounded quarterly which is required to triple his money in 15 years? How much would the interest rate need to be if it is compounded continuously?
Answer: We find $j_4 = 0.0739155$ and $\delta = 0.0732408$.

24. Marie invested 150€ at 0.1% monthly interest compounded continuously. Find the amount accumulated at the end of 4 years.
Answer: $S = 150e^{0.001\times12\times4} = 157.38$.

25. What is the formula for calculating projects with

    25.1. simple interest rate?

    25.2. compounded interest rate?

    25.3. continuously compounded rate?

    Write each formula twenty times.

26. Jacky invested 1,000€ at 8% interest compounded continuously. Find the amount accumulated at the end of 7 years and three months. What if interest

is compounded quarterly?
Answer: Seven years and three months give $7 + 3/12 = 87/12$. Continuous compounding gives $S = 1000e^{0.08\times 87/12}$, or quarterly gives $S = (1 + 0.08/4)^{4\times 87/12}1000$.

27. Joakim wants to save for a new car. Since the year 2012 he saved 5,000€ every Christmas. The bank gives him an interest rate of 4%. What is the total amount of money that Joakim saved after 4 years?
Answer: The savings after four years from the first year are $S_1 = 5000(1 + 0.04)^4$, from the second $S_2 = 5000(1 + 0.04)^3$, from the third $S_3 = 5000(1 + 0.04)^2$, and from the fourth $S_4 = 5000(1 + 0.04)$. So the total savings amount to $S = S_1 + S_2 + S_3 + S_4$.

28. Assume your salary in two years from now is 40,000€. Given an interest rate of 4% compounded monthly, how much would your salary be worth today, and how much would it be worth in 18 months from now?
Answer: We first calculate $40000 = (1+0.04/12)^{12\times 2}P$ and solve for P, which gives $P = 36,929.6$. In 18 months from now this becomes $S = 36,929.6(1 + 0.04/12)^{12\times 18/12}$, which is $S = 39,209.3$.

29. You need 25,000€ to buy a car. Assume the price of the car stays constant. You have 15,000€ now which you can invest at a 3% interest rate compounded quarterly, or at a 2.5% interest rate compounded continuously. How long does it take with either of these two investments to accumulate the money you need for the car?
Answer: For quarterly compounding it takes $t = 17.0913$ years, which is 17 years and around 33 days. For continuous compounding it takes $t = 17.0275$, which is 17 years and 10 days.

30. A business partner borrows 100,000€ from you for one year and three months. You want him to return 125,000€ to you. What would be the annual interest rate if interest

    30.1. compounded monthly?
    Answer: Compound interest, one year and three months is $t = 1+3/12 =$

15/12, so we calculate $125,000 = 100,000\left(1 + j_{12}\frac{15}{12}\right)$.

30.2. compounded continuously?

Answer: $125,000 = 100,000e^{\delta\frac{15}{12}}$. Using logs with base e we get $\ln 1.25 = \frac{15}{12}\delta$. Solving for δ gives us then the answer.

31. Assume you want to borrow 1,000,000€ for ten year and three months. You know that at the end of that period you have to return 1,250,000€ to the bank. What would be the annual interest rate if interest

31.1. compounds quarterly

Answer: $1,250,000 = 1,000,000\left(1 + j_4\frac{123}{12}\right)$.

31.2. compounds continuously.

Answer: $1,250,000 = 1,000,000e^{\delta\frac{123}{12}}$.

32. You want to undertake an investment of 100,000€. After 5 years you expect to gain 140,000€. You have several banks that could give you a loan. What is the maximum interest rate that you would expect to pay?
Answer: The highest interest rate would come from annual compounding and would solve $140,000 = 100,000(1 + j_1)^5$, which is $j_1 = 0.06961$.

33. A company wants to undertake an investment. It will face a cash outflow of 1 million euro at the end of 2018, and expects a cash inflow of 300,000€ at the end of 2019, 350,000€ at the end of 2020, 200,000€ at the end of 2021 and 140,000€ at the end of 2022. If the interest rate is 7%, will this be a good investment for the company at the end of 2022?
Answer: We calculate $-1,000,000 + 300,000/(1 + r) + 350,000/(1 + r)^2 + 140,000/(1 + r)^3$ with $r = 0.07$.. It will face a loss of $-299,641$€, so no.

34. Tarzan borrows 40,000€ from his friend Robinson. He wants to repay this loan in two annual installments, one and two years years after he took out the loan. Each of these repayments is 21,512.2€. What is the interest rate on this loan?

Answer: Here you need to be a bit innovative. So we know that the total repayments should be $40,000$ plus the interest. However, as he repays some of the loan, then in the next period he won't need to pay so much interest from the previous period. So, Tarzan borrows 40000, and after one year he needs to repay 40000 plus interest, which is $40000(1+r)$. Then Tarzan repays 21,512.2€, so his outstanding debt is $40,000(1+r)-21,512.2$. After the next year he again pays interest on the remaining amount which is $\Big(40,000(1+r)-21,512.2\Big)(1+r)$, but he repays again (so we have now $\Big(40,000(1+r)-21,512.2\Big)(1+r)-21,512.2$). Then he should have repaid everything, so we know that this should equal zero, thus $\Big(40,000(1+r)-21,512.2\Big)(1+r)-21,512.2=0$. Then using the Characteristic Polynomial and solving for r we find that $r=0.05$.

4 Comparing interest rates

1. Find the quarterly compounded annual interest rate that is equivalent to a monthly compounded monthly interest rate of 0.3%. Also, find the annually compounded quarterly interest rate that is equivalent to a continuously compounded interest rate of 1%.

   Answer: In the first part we solve $\left(1+\frac{j_1}{4}\right)^4=\left(1+\frac{j_{12}\times 12}{12}\right)^{12}$. This gives $j_4=3.61\%$
   In the second question we solve $1+4i=e^{0.01}$. This gives $i=0.25\%$

2. Rob invested 12,000€ at a rate of 10% annual interest compounded monthly. Find the value of his investment after 6 years. What is the equivalent annually compounded interest rate?
   Answer: $S=21,811.1$€
   $r_e=10.47\%$.

3. Which of the following is more attractive?

   3.1. 5.2% annual rate at annually compound interest

   3.2. 5% annual rate at monthly compound interest

3.3. They are the same.

Answer: The AER for the annual rate at monthly compound interest is 5.116% ($< 5.2\%$).

4. Calculate the per interest period (proportional) and the annual equivalent interest rate for each of these cases:

   4.1. An annual interest rate of 3% compounded quarterly
   Answer: $i = 0.0075$ and $r_e = 0.03034$.

   4.2. A monthly interest rate of 0.4% compounded daily
   Answer: $i = 0.004 * 12/360 = 0.000133$ and $r_e = 0.04917$.

   4.3. A quarterly interest rate of 4% compounded annually
   Answer: $i = 0.04 * 4 = 0.16$ and $r_e = 0.16$.

   4.4. A daily interest rate of 0.01% compounded monthly
   Answer: $i = 0.0001 * 360/12 = 0.003$ and $r_e = 0.0366$.

5. Which of the following rates is more attractive?

   - 5.2% compounded annually
   - 5% compounded quarterly
   - 4.8% compounded monthly
   - 4.5% compounded continuously
   - They are the same.

   Answer: We calculate the AER for each. The AER for 5.2% compounded annually is 5.2%; for 5% compounded quarterly is 5.09%; for 4.8% compounded monthly is 4.907%; for 4.5% comp. cont. is 4.603%.

6. Which of the following rates is more attractive?

   - 0.433% monthly interest rate compounded quarterly
   - 0.015% daily interest rate compounded monthly
   - They are the same.

Answer: 1.05298% < 1.05536%.

7. Find the monthly compounded interest rate that is equivalent to a quarterly compounded monthly interest rate of 3%.
Answer: $j_{12} = 0.34971$.

8. Find the daily compounded monthly interest rate that is equivalent to a continuously compounded interest rate of 10%.
Answer: $j_{360}/12 = 0.00833$.

9. Find the proportional and the equivalent interest rates for each of the subsequent cases.

   9.1. Annual interest rate of 3.5%, what is the rate at monthly compounding?
   Answer: Proportional: 0.2917%
   Equivalent: 0.287%

   9.2. Quarterly interest rate of 1.2%, what is the rate at annual compounding?
   Answer: Proportional: 4.8%
   Equivalent: 0.048

   9.3. Annual interest rate of 4%, what is the rate at semi-annual compounding?
   Answer: Proportional: 2%
   Equivalent: 0.0396

5 Annuities

1. You rent an apartment in Paris. At the end of each month you have to pay 1,200€ rent. Given that the monthly compounded interest rate is 4%, how much is the total amount of rent worth after four years?
Answer: This is annuities, so $S = \frac{(1+i)^n-1}{i}R$, where $R = 1200$, $i = 0.04/12$ (as it is the monthly compounded interest rate and annual is not specified then it is the annual rate), $n = 12 \times 4$, so we get $S = \frac{(1+0.04/12)^{48}-1}{(0.04/12)}1200$.

2. What is the monthly payment on a rent contract that, in 20 years, totals 350,000€ (inclusive interest), with an annual interest rate of 4.5% compounded annually?
   Answer: This is a general annuity, so we need to first calculate the equivalent rate. $1+0.045 = (1+i)^{12}$. Solving for i we get $i = (1.045)^{1/12} - 1 = 0.003675$. Then we substitute this into the equation for annuity, $S = \frac{(1+i)^n - 1}{i} R$, giving $350,000 = \frac{(1+0.003675)^{20\times 12} - 1}{0.003675} R$. Then we solve for R.

3. You want to buy a Tesla Model S which will cost you 92,000€. How much money do you need to lay aside each month during the next four years so that you can afford this car at an annual interest rate of 3.5% compounded annually?
   Answer: This is a general annuity, so we need to first calculate the equivalent rate. $1+0.035 = (1+i)^{12}$. Solving for i we get $i = (1.035)^{1/12} - 1 = 0.0028709$. Then we substitute this into the equation for annuity, $S = \frac{(1+i)^n - 1}{i} R$, giving $92,000 = \frac{(1+0.0028709)^{4\times 12} - 1}{0.0028709} R$. Then we solve for R giving $R = 1,790.38$.

4. You have inherited 20,000€ and you wish to purchase a contract that will provide you a steady income for the next 10 year. Banks are paying a 4.3% annual interest rate. How much would you be able to receive on a yearly basis?
   Answer: The 20,000€ is money that you have now, and if you place it into the bank then it accumulates interest. However, not the total amount of the money will accumulate interest during all periods, as you also withdraw money in regular intervals. Let's see how this works. At the beginning of the first period you place the 20,000€ into an account. At the beginning of the year you then have $20,000(1+i)$, where $i = 0.043$, but you also withdraw a certain amount, let's call that R. Thus, you'll have $20,000(1+i) - R$. At the beginning of the next period you earn interest on this money and you withdraw an amount R again, giving $(20,000(1+i) - R)(1+i) - R$. This gives us $20,000(1+i)^2 - R - R(1+i)$. Assume we continue this for the next ten years, then we would get $20,000(1+i)^{10} - R - R(1+i) - ... - R(1+i)^9$. The term $20,000(1+i)^{10}$ gives us the future value of the money inherited, while the terms $R + R(1+i) + ... + R(1+i)^9$ is our formula for an ordinary simple annuity.

   So the steps are as follows. We first need to calculate the future value of the money that you inherited. That's compound interest, and it gives us $S = 20000(1 + j_1)^t$, where $t = 10$ and $j_1 = 0.043$. This is $S = 30,470$. So that will be the total future value of the annuity. Thus we now solve $30,470 = \frac{(1+0.043)^{10}-1}{0.043}R$, which yields $R = 2,502.78$.

5. You would like to go on a vacation but cannot afford the total cost of 3,000€. Money is valued at a 5% interest rate compounded annually. You can only save 350€ per month, so how long do you need to save in order to be able to enjoy this vacation?
Answer: First we need to use equivalent rates to calculate the monthly rate. This is $1+0.05 = (1+i)^{12}$, giving $i = (1.05)^{1/12} - 1 = 0.004074$. Thus, we now have $3000 = 350\frac{(1+0.004074)^t-1}{0.004074}$. Solving for t we get $\log\left(\frac{3000\times0.004074}{350} + 1\right) = t\log(1+0.004074)$, giving $t = 8.44232$, which is eight months and $0.44\times30 = 13$ days.

6. For the past 12 years you have been saving 1,000€ at the end of the year. In the first four years the annually compounded annual interest rate was 2%. During the next three years you received an annually compounded annual interest rate of 4%. For the last years the interest rates reduced to 1.5%. How much money do you have now?
Answer: $S = 1000\frac{1.02^4-1}{0.02}(1 + 0.04)^3(1 + 0.015)^5 + 1000\frac{1.04^3-1}{0.03}(1 + 0.015)^5 + 1000\frac{1.015^5-1}{0.015}$ which yields $S = 14,630.6$

7. At the end of each year you saved 1,200€ because you want to buy a car. You have been doing that for 10 years now. During the first two years the annually compounded interest rate was 5%. Then you noticed that in another bank you get better rates and for thee years you received an annually compounded rate of 6%. A new regulation was then put in place that reduced the rates for all banks to 2.5%. How much money do you now have in your bank account?
Answer: Simple ordinary annuity. $R = 1200$, $t_1 = 2$ and $r_1 = 0.05$; $t_2 = 3$ and $r_2 = 0.06$, $t_3 = 5$ and $r_3 = 0.025$. Formula for annuity is $S = R((1 + r)^t - 1)/r$. Then need to combine with the fact that the first periods also accumulate compounded interest. Thus the complete formula becomes $S = S_1(1 + 0.06)^3(1 + 0.025)^5 + S_2(1 + 0.025)^5 + S_3$. Yields $S = 13,944.84$.

8. You buy a new taxi for 40,000€. As a taxi driver you will drive this car for three years and each year you expect to make 15,000€. At the end of the three years you believe your car is worth nothing any more. What is the internal rate of return on this investment? Is this investment worthwhile at a rate of $r = 0.05$?
   Answer: Using the formula for the internal rate of return we have $0 = -40000 + 15000/(1+r) + 15000/(1+r)^2 + 15000/(1+r)^3$. We write this as $40000 = \frac{(1-(1+r)^{-3}}{r}15000$. Then we solve this recursively by trying different solutions for r. So in our calculator we write $\frac{(1-(1+r)^{-3}}{r}15000$, with specific trials for r, such that as a solution we obtain 40,000€. Let's say our first try is $R = 0.03$, which gives $42,429.2$. That's more than 40000 so we need to calculate with a higher IRR. Let's try $r = 0.1$. This gives 37,302.8 which is too low. Let's try $r = 0.05$, which gives 40,848.7. Still a bit too high, so let's try a slightly larger IRR, $r = 0.06$. This gives 40,095.2. A slightly larger $r = 0.065$ gives 39,727.1. After some more trials we converge to $r = 0.0612$ which is 40,006.3. We know that the IRR is 6.12%, which is larger than 5%, so this investment would be worthwhile.

9. You invest into a new hotel which will cost you 10 million euro. You believe that annually you will make a profit of 1 million euro during the next 5 years. Then you want to sell the hotel again and you hope to be able to sell it for 5 million euro. What is the internal rate of return on this investment? Is this investment worthwhile at a rate of $r = 0.05$?
   Answer: Using the formula for the internal rate of return we have $0 = -10,000,000 + 1,000,000/(1+r) + 1,000,000/(1+r)^2 + 1,000,000/(1+r)^3 + 1,000,000/(1+r)^4 + 1,000,000/(1+r)^5 + 5,000,000/(1+r)^5$. In this case we can't use the other formula the periodic returns are not equal over time. Recursively trying different solutions for r we find that $r = 0.055$. Thus, for any interest rate which is less than $r = 0.055$ we know that the investment will be profitable.

6 Loan Amortization

1. If a mortgage is amortized over 20 years at an annual interest rate of 7% compounded monthly and monthly payments of 250€, then what is the original

value of the mortgage?
Answer: We have $R = \frac{iP}{1-(1+i)^{-n}}$, where $n = 20 \times 12$, $i = 0.07/12$, $R = 250$, then $\frac{250}{0.07/12}\left(1 - (1 + 0.07/12)^{-20\times12}\right) = P$, giving $P = 32,245.6$.

2. You want to get a mortgage for 250,000€ with monthly payments at an annual interest rate of 4.5% compounded annually. You can afford to pay back 1,200€ every month. How long would it take to pay off the mortgage? Bonus question: What happens if the monthly repayment is 700?
Answer: First apply equivalent rates to convert the annually compounded annual interest rate into a monthly compounded monthly one. This is $1 + 0.045 = (1 + i)^{12}$, where we get $i = 0.00367481$. Then we use this in the loan amortization formula, where we will have $1200 = \frac{0.00367481\times250000}{1-(1+0.00367481)^{-t\times12}}$. Simplifying we get $1 - \frac{0.00367481\times250000}{1200} = (1 + 0.00367481)^{-t\times12}$, then taking logs, using the Potency Rule, simplify and solve we get $t = 32.6068$ years (32 years and 219 days).

3. Amortize a mortgage of 250,000€ at an interest rate of 4% where the mortgage is paid back during 4 years with payments at the end of each year. Draw an amortization table.
Answer: $R = \frac{iP}{1-(1+i)^{-n}}$, where $P = 250000$, $i = 0.04$, $n = 4$, gives $R = 68,872.5$.

4. Assume you work in a bank and someone wants to take out a loan of 200,000€ for 10 years. You will charge this person an annual interest rate of 2% compounded annually. How much does this person then need to pay back each quarter to fully repay the loan? (Hint: be careful with the interest rate and the quarterly repayment)
Answer: First calculate equivalent rates: $1 + 0.02 = (1 + i)^4$, giving $1.02^{1/4} - 1 = i$. Then use the solution of this i in the formula for loan amortization, $R = \frac{iP}{1-(1+i)^{-n}}$, where $n = 4 \times 10$, i is the solution calculated previously, $P = 200000$. This gives $R = 5,525.06$.

5. Tarzan borrows 40,000€ from his friend Robinson. He wants to repay this loan in two annual installments, one and two years years after he took out the

loan. Each of these repayments is 21,512.2€. What is the interest rate on this loan?
Answer: This is the same as the question in the compound interest section, but now you know you can answer this with the loan amortization formula. So we have $21,512.2 = \frac{i40{,}000}{(1-(1+i)^2}$, giving $i = 0.05$.

6. Suppose you want to borrow money to buy a house. You are considering a 15-year or a 30-year loan. The lender offers different interest rates, reflecting the differences in risks of shorter-term and longer-term lending. For the 15-year loan, the annual rate is 6.25% (compounded monthly). For the 30-year loan, the annual rate is 6.75% (compounded monthly).

   6.1. If you borrow 150,000€, what would your monthly payments be for each loan? Which loan is better? (several views possible)
   Answer: For the 15 year year loan we'd calculate $R = \frac{0.0625/12 \times 150000}{(1-(1+0.0625/12)^{-12\times15}}$, giving $R = 1,286.13$. For the 30 year loan, we'd have $R = \frac{0.0675/12 \times 150000}{(1-(1+0.0675/12)^{-12\times30}}$, giving $R = 972.9$. Which loan is better depends on what you would do with the difference between $1,286.13$€ and 972.90€ for the first 15 years. Would it be worth having that extra 313.23€ for fifteen years and then paying 972.90€ for another fifteen years? Your call - depending perhaps on your personal preference for immediate gratification.

7. You want to take out a loan of 20,000€ that you plan to pay back fully at an annual interest rate of 5% (compounded monthly) during the course of the next three years. How much do you then need to pay back each month? How much do you need to repay in total?
Answer: Loan amortization, formula is $P = R(1 - (1 - r)^{-t}/r$. $P = 20,000$; $r = 0.05/12$; $t = 3 \times 12 = 36$. Gives $R = 599.39$ and total $36 \times R = 21578.04$.

8. You owe a local gambling house 60,000€. It charges a terrible interest rate of 10%. Every year someone from the gambling house comes to you and collects a (constant) amount of the debt. Your problems started in the year 2019, and the gambling house told you that you'll have to have everything repaid by 2030.

8.1. How much do you have to repay every year?
Answer: $R = \frac{iP}{1-(1+i)^{-n}}$, where $i = 0.1$, $P = 60000$, and $n = 11$, giving $R = 9,237.79$.

8.2. How much debt do you still have outstanding by 2025?
Answer: This requires the formula for debt outstanding, which is $X_m = \left(P - \frac{R}{i}\right)(1+i)^m + \frac{R}{i}$. In period $m = 6$ this yields $X_6 = 35,018.5$.

8.3. How much *of your debt* will you have repaid by 2025?
Answer: Here you need to be careful. Your original debt was $P = 60000$. But in each period you pay interest which adds to your total debt. So you need to calculate your total debt, which will be the sum of your payments. That's $11 \times 9,237.79 = 101,616$. So by 2025 you will have paid $6 \times 9,237.79 = 55,426.7$.

8.4. How much interest in total will the gambling house have received in 2030?
Answer: We know you pay in total $11 \times 9,237.79 = 101,616$. So this total minus your initial debt ($P = 60000$) gives your total interest payments, yielding $101,616 - 60000 = 41,616$€.

8.5. How large would have been your debt if you had paid the whole sum back in 2030? What explains the difference?
Answer: If you had not paid back R every year, then after 11 years your total debt would have been $S = P(1 + \frac{j_m}{m})^{mt}$, where $m = 1$, $j_1 = 0.1$, $P = 60000$, $t = 11$, giving $S = 171,187$. This is around $70,000$€ more and the difference is due to the fact that your annual repayments reduce your debt outstanding and thus reduce your annual interest fees.

9. You own a patent with a duration of 25 years. You spent 20,000€ on the design development. Given an interest rate of 3.3%, how large would be the (constant) amount that you would have place in your annual balance sheet each year so that the patent gets amortized after the 25 years?
Answer: Loan amortization, so $R = \frac{iP}{1-(1+i)^{-n}}$, where $i = 0.033$, $P = 20000$, and $n = 25$, giving $R = 1,187.29$.

10. Assume you take out a loan of 10,000€ at an annual interest rate of 5% that you plan to repay quarterly over the next three years.

10.1. What would be the quarterly interest rate that is equal to the annual one?
Answer: Equivalent rates, so you calculate $(1+i)^4 = 1+0.05$, which is equal to $i = 1.05^{1/4} - 1 = 0.01227$.

10.2. How much would you need to repay every quarter?
Answer: Loan amortization, so $R = \frac{iP}{1-(1+i)^{-n}}$, where $i = 0.01227$, $P = 10000$, and $n = 4 \times 3 = 12$, giving $R = 901.28$.

10.3. Draw the first four lines of the amortization table.

10.4. How much debt is still outstanding after two years?
Answer: The debt outstanding is $X_m = \left(P - \frac{R}{i}\right)(1+i)^m + \frac{R}{i}$, which for $m = 4 \times 2 = 8$, the rest is the same as above, giving $X_8 = 3,497.21$.
For the amortization table, remember that the debt outstanding is calculated by $X_m = \left(P - \frac{R}{i}\right)(1+i)^m + \frac{R}{i}$ (another way is to calculate, in each period, $X_{t-1}(1+i) - R$), the interest at time t is $X_{t-1}i$, and the principal repaid is $R - X_{t-1}i$, with $X_0 = P$. Then the amortization table is given by

Period	R	debt outstanding	interest	Principal repaid
1	901.28	9,221.42	122.7	778.58
2	901.28	8,433.29	113.15	788.13
3	901.28	7,635.48	103.47	797.8
4	901.28	6,827.89	93.69	807.6

10.5. What is the annual rate that is equivalent to the quarterly rate?
Answer: We of course know this from above, it is 0.05.

11. A debt of 10,000€ is amortized by making equal payments at the end of every six months for three years, and interest is 6% compounded semi-annually.

11.1. Determine the payments.
Answer: Loan amortization, so $R = \frac{iP}{1-(1+i)^{-n}}$, where $i = 0.06/2 = 0.03$, $P = 10000$, and $n = 3 \times 2 = 6$, giving $R = 1,845.98$.

11.2. Construct an amortization table. Answer: For the amortization table, remember that the debt outstanding is calculated by $X_m = \Big(P -$

$\left.\frac{R}{i}\right)(1+i)^m + \frac{R}{i}$ (another way is to calculate, in each period, $X_{t-1}(1+i) - R$), the interest at time t is $X_{t-1}i$, and the principal repaid is $R - X_{t-1}i$, with $X_0 = P$. Then the amortization table is given by

Period	R	debt outstanding	interest	Principal repaid
1	1,845.98	8454.02	300	1545.98
2	1,845.98	6861.66	253.62	1592.36
3	1,845.98	5221.53	205.85	1640.13
4	1,845.98	3532.2	156.646	1689.33
5	1,845.98	1792.18	105.97	1740.01
6	1,845.98	0	53.77	1792.21
Totals	11075.9		1075.9	10000

7 Past exam questions

In this section you can test yourself with some past exam questions. Try not to look at the answers unless you either believe you solved the question, or you see absolutely no way to find a solution.

1. You have 2,100€ in your bank account. At a 2% annual interest rate, how much did you deposit 111 days prior to this?
   Answer: $P = \frac{2100}{1+0.02\frac{111}{360}}$, giving $P = 2087.13$.

2. A company has a bill of exchange with 120,000€ due on the 12th April. On the 2nd of February it needs liquidity of 110,000€ and it wants to sell this bill to a private investor on. How much would the interest rate be? If the private investor could instead invest his money at an annual interest rate of 5%, what would the investor prefer?
   Answer: $120000 = 110000(1 + r70/360)$, gives $r = 0.4675$. Clearly, $r = 0.4675 > 5\%$.

3. A company wants to borrow 120,000€ from its bank for an investment. It will repay this loan in two annual instalments respectively one year and two years after the loan. Each of these payments is 61,250€. What is the interest rate on this loan?
   Answer: $(120000(1 + r) - 61250)(1 + r) - 61250 = 0$, gives $r = 0.0138571$.

4. You want to set up a start-up company at the beginning of 2020. For this you need to borrow 250,000€. At the end of the 2021 you expect to receive a revenue of 60,000€, at the end of 2022 you hope for a revenue of 110,000€, and at the end of 2023 you expect a final cash inflow of 130,000€. These inflows and outflows occur in the same account at a bank that works with an annual interest rate of 10%. Is this investment profitable?
   Answer: $-250000+60000/((1+0.1)^2)+110000/((1+0.1)^3)+130000/((1.1)^4) = -28976.8$, not profitable.

5. You place 5200€ into a bank account at 3% interest. How much do you have in your bank account after 54 days?
   Answer: $S = (1 + 0.03x54/360)5200 = 5224.3$.

6. A banker is in the possession of a bill of exchange of 12,200€ to be exchanged on the 30th December. On the 13th October he sells this bill of exchange to a bank. This bank calculated with an interest rate of 6%. How much will the bank be willing to give the banker for this bill of exchange?
   Answer: $12200 = (1 + 0.06x77/360)P$, so $P = 12200/(1 + 0.06x77/360) = 12,045.4$.

7. A craftsman borrows 20,000€ from his bank. He will repay this loan in two annual instalments respectively one year and two years after the loan. Each of these annuities is 10,641.93€. What is the interest rate on this loan?
   Answer: $(20,000(1+rt)-P)(1+rt)-P = 0$ Gives $(20000(1+r)-10641.93)(1+r) - 10641.93 = 0$ Solving for r gives $r = 0.0425$.

8. A company is considering an investment with the following financial elements: Cash outflow of 180,000€ at the beginning of 2020, cash inflow of 60,000€ by the end of 2020, cash inflow of 80,000€ at the end of the year 2021, cash inflow of 70,000€ in late 2022. No other cash flow is expected. Is this investment profitable at 9%?
   Answer: $-180000+60000/(1+r)+80000/(1+r)^2+70000/(1+r)^3$ For $r = 0.09$ this yields $-3566.89 < 0$, so no it is not profitable.

9. On the 20th October 2019 you had 12,000€ in your bank account. If the interest rate is 1.5%, how much did you have on the 2nd May 2019?
   Answer: Simple interest, $12000 = (1 + 0.015\frac{168}{360})P$, giving $P = 11,916.6$.

10. You have 16,000€ in your bank account. If the monthly interest rate compounded quarterly is 1%, how much will you have in your account in five years and two months?
    Answer: $S = (1 + \frac{0.01\times12}{4})^{4\times(5+2/12)}16,000$, giving $S = 29,472.9$.

11. If you borrow 8,000€ for five years at an annual interest rate of 5%, how much do you need to repay at the end of every year?
Answer: $R = \frac{0.05 \times 8000}{1-(1+0.05)^{-5}}$, giving $R = 1,847.8$.

12. What is the interest rate if you borrow 120,000€ on the 5th of April 2019 and you have to give back 125,000€ on the 4th December?
Answer: simple interest. $r = 0.0628$

13. You borrow 80,000€ at a monthly compounded, quarterly interest rate of 0.5%. You will have to return 100,000€ at a later date. How long is the duration of this loan?
Answer: compound interest. You solve $100,000 = 80,000\left(1 + \frac{0.005 \times 4}{12}\right)^{12 \times t}$.
Yields $t = 11.1665$, which is 11 years and just about 60 days.

14. You borrow 10,000€ for 10 years at an annually compounded, annual interest rate of 3.5%. You repay this quarterly. How much do you need to repay at the end of every quarter?
Answer: Loan amortization. First need to calculate the per period interest rate. For that we use equivalent rates. We use $1 + 0.035 = (1 + \frac{j_4}{4})^4$. This gives $j_4 = 0.0345498$. That's still the annual interest rate but quarterly compounded. Now we need to calculate the quarterly rate, or the per period rate, which is $j_4/4 = 0.00863745$. This we now use in our loan amortization formula, giving $R = \frac{0.0086374510{,}000}{1-(1+0.00863745)^{-4 \times 10}}$, which gives $R = 296.74$.

15. 1. An investor needs short-term financing of 2 million euros for 10 days. His bank offers him a spot loan, at an annual interest rate of 4%. What interest will he have to pay for this financing?
Answer: Approach: simple interest
Formulas: $I = rtP$
(un)knowns: $I = I$; $r = 0.04$; $t = 10/360$; $P = 2000000$
Solution: $I = 0.04 * 10/360 * 2000000$, giving $I = 2222.22$.
Answer: The total interest on this loan is 2,222.22 euro.

16. 2. A merchant received a bill of exchange of €100,000 from one of his customers. This bill of exchange expires on March 21, 2021. On 12 January 2020, he gives this bill of exchange at a discount to his bank. His bank charges a continuously compounded, annual interest rate of 3%. How much will she give to the merchant in exchange for this bill of exchange?
Answer: Approach: continuously compound interest
Formulas: $S = e^{(}\delta t)P$
(un)knowns: $S = 100000$, $\delta = 0.03$, $P = P$, $t = (360 + 60 + 9)/360 = 1.191667$
Solution: $100000 = e^{(}0.03 * 1.191667)P$.
Answer: The bank will give 96,488.15 euro to the merchant.

17. 3. To build up capital, a sum of 5000 euros is invested for 6 years at the end of each year in an account yielding a 2.5% annual interest rate. What will be the total capital on the day of payment of the last annual instalment?
Answer: Approach: ordinary simple annuity
Formulas: $S = ((1 + i)^n - 1)R/i$
(un)knowns: $S = S$, $i = 0.025$; $n = 6$; $R = 5000$
Solution: $S = ((1 + 0.025)^6 - 1)5000/0.025$
Answer: The total capital will be 31,938.68 euro.

18. 4. Mr Blue buys a car worth €8,250. He takes out a consumer loan with his bank. This loan is repayable at an interest rate of 4.80%. The loan is taken out on 01/03/2020, and is repayable by 20 constant monthly instalments (the first maturing on 01/04/2019). a) Calculate the per period interest rate. b) What is the amount of each monthly repayment? c) Present the first two lines of the amortization table of the loan. d) What is the outstanding debt, just after the 10th monthly payment?
Answer: a. Calculate the per period interest rate.

Approach: calculating equivalent interest rates
Formula: $1 + r_e = (1 + i)^12$
(un)knowns: $r_e = 0.048$; $i = i$
Solution: $1 + 0.048 = (1 + i)^12$
Answer: the per period interest rate is $i = 0.00391461$.

b. What is the amount of each monthly repayment?

Approach: Loan amortization
Formula: $P = (1 - (1 + i)^(- T))R/i$
(un)knowns: $P = 8250$; $i = 0.00391461$; $T = 20$; $R = R$;
Solution: $8250 = (1 - (1 + 0.00391461)^(- 20))R/0.00391461$.
Answer: The monthly repayments are 429.67 euro.

d. What is the outstanding debt, just after the 10th monthly payment? Approach: debt outstanding
Formula: $X_t = (P - R/i)(1 + i)^t + R/i$
(un)knowns: $X_t = X_t$; $P = 8250$; $R = 429.67$; $i = 0.00391461$; $t = 10$;
Solution: $X_10 = (8250-429.67/0.00391461)(1+0.00391461)^10+429.67/0.00391461$
Answer: The debt outstanding after 10 months is 4205.52 euro.

19. 5. A company is thinking of buying a new machine, costing €500,000, to lower manufacturing costs. The annual savings thus achieved are estimated at €120,000 for each of the 6 years of the estimated lifetime of this equipment. As manufacturing processes are rapidly evolving in this field, it is estimated that after 6 years, the equipment will be out of date and will have to be replaced. Its value at the end of the 6 years of use is considered nil. a) Is this investment profitable at a 10% interest rate? b) What is the internal rate of return on this investment?
Answer: a. Is this investment profitable at a 10% interest rate?

Approach: ordinary simple annuity, Net Present Value
Formulas: OSA: $S = ((1 - i)^n - 1)R/i$, NPV: $B = S/(1 + i)^n - C$
(un)knowns: $S = S$; $i = 0.1$; $R = 120000$; $B = B$; $C = 500000$; $n = 6$
Solution: $S = ((1 - 0.1)^6 - 1)120000/0.1 = 925873$. That's the future value.
Then $B = 925873/(1 + 0.1)^6 - 500000 = 22631.2$
Answer: the total savings will be 22,631.2 euro. The investment is profitable at a 10% interest rate.

b. What is the internal rate of return on this investment?

Approach: Internal rate of return
Formula: $C = (1 - (1 + i)^(- T))R/i$
(un)knowns: $i = I$; $C = 500000$, $T = 6$; $R = 120000$;
Solution: $500000 = (1 - (1 + i)^(- 6))120000/i$. We try $i = 0.1$, gives RHS = 522631 which is slightly larger than 500000. Thus we try a larger i, take $i = 0.12$. This gives RHS = 493369, which is slightly smaller. So we try a

smaller i, etc. An exact solution gives $i = 0.115305$.
Answer: The internal rate of return is 11.5305%.