Université Paris 1 Panthéon - Sorbonne
U.F.R des Sciences Économiques

Essays on Environmental Policy under Catastrophic Event Uncertainty

Can Askan Mavi

Thèse présentée et soutenue publiquement
à Paris le 26 Septembre 2018
en vue de l'obtention du grade de
DOCTORAT EN SCIENCES ÉCONOMIQUES
de l'Université Paris 1 Panthéon-Sorbonne

Directeur de thèse:

Antoine d'Autume, Professeur des universités, Université Paris 1 Panthéon-Sorbonne

Composition du jury:

Antoine Bommier, Professeur, ETH Zürich
Mouez Fodha, Professeur des universités, Université Paris 1 Panthéon-Sorbonne
Katrin Millock, Chargée de Recherche CNRS, Université Paris 1 Panthéon-Sorbonne
Lionel Ragot, Professeur des universités, Université Paris-Nanterre
Ingmar Schumacher, Professeur, IPAG Business School

This thesis has been carried out jointly
at the Centre d'Economie de la Sorbonne, Maison des Sciences Economiques,
106-112 Boulevard de l'Hôpital, 75647 Paris Cedex 13, France
and at Paris-Jourdan Sciences Économiques - UMR8545,
48, Boulevard Jourdan 75014 Paris, France.

Cette thèse a été préparée conjointement
au Centre d'Economie de la Sorbonne, Maison des Sciences Economiques,
106-112 Boulevard de l'Hôpital, 75647 Paris Cedex 13, France
et à Paris-Jourdan Sciences Économiques - UMR8545,
48, Boulevard Jourdan 75014 Paris, France.

To my grandfather, Adil...

Acknowledgements

Writing a Ph.D. thesis is like learning to fly. When you are a little baby bird, you look at the sky and see other birds flying beautifully. People who love documentaries know that every baby bird stays on the nest until they grow up and their wings get bigger. These baby birds also check frequently if they can fly or not by flapping their wings. Of course, first attempts always fail. During this interesting process, mother or father bird show to baby birds how to fly by flying from one point to another. I am deeply grateful (and always will be!) to Antoine d'Autume for showing me how to fly. I do not know how many times I fell down and how many more times I will fall again but every time I found myself on the ground, Antoine d'Autume helped me to stand up and to try again and again. I learned so much from him and I always enjoyed discussing and sharing my ideas with him. Now, he says that I grew up sufficiently to fly by myself. I promise I will do my best!

I would like to thank Antoine Bommier, Mouez Fodha, Katrin Millock, Lionel Ragot and Ingmar Schumacher for giving their precious time to read this thesis and for giving very insightful comments to enhance the quality of this work.

It goes without saying that a Ph.D. candidate always benefits from the comments of many senior researchers. I express my gratitude to Bertrand Wigniolle, Mouez Fodha, Katrin Millock, Katheline Schubert, Mireille Chiroleu-Assouline, Hélène Ollivier, Fanny Henriet, Lionel Ragot, Stéphane Zuber, Hippolyte d'Albis, Fabien Prieur, François Salanié, Ingmar Schumacher, Stefano Bosi, Pierre Lassere, André Grimaud, Gilles Lafforgue, Jean-Marc Bourgeon.

Research is a wild world and it would be difficult to fight against difficulties without people sharing similar problems. I had a perfect first year in Ph.D. thanks to "Environmental Economics team". I thank especially Diane, Mathias (no way!), Stefanija, Lorenzo Cerda, Thais, Hamzeh and Baris with whom I had many wonderful moments. I am also grateful to all my friends in office 221 at MSE: Mathieu, Matthieu (thanks for the Strokes night at Supersonic!), Guillaume, Salim, Zeinap, Hélène. Of course, my special thanks go to my friends Rizwan for being the best of "Pachas" and Anna for cheering the office 221!

When we were moving from MSE to Jourdan, I was thinking that I would never have another office 221! However, I enjoyed so much being a member of R4-64 at Jourdan. You can not imagine an office with 8 young "macro boys", all of them with a beard, making funny jokes all the time (sometimes inappropriate!). I thank Marco, Stefano, Emanuele (I always learned something fancy from you, Emanuele!), Jaime (patron), Mehdi (I always miss your unexpected funny jokes), Yassine and Normann (thanks for your refined jokes and old French words!).

I would like to express my gratitude to my Ph.D. brothers: Hamzeh (thank you so much for your very precious friendship!), Moutaz (thanks a lot for always motivating me), Yassine (we are brothers since L3, thanks so much for your sincere advices!) and Baris, I think I could not have started this thesis without your help. Thanks a lot for your support and for sharing your huge Mathematica knowledge!

I am grateful to two wonderful people: Armagan and Okay. Okay is the boss! When I write "the man" on Google, it gives Okay as a result (sorry for the very "Turkish" idiomatic joke!). Well, do not expect from me to tell all our jokes here! Thanks for your unfailing support!

I am also thankful to many people both at MSE and at Jourdan: Ilya (thanks for all these "weird" jazz albums that maybe nobody knows on the planet!), Irene, Emna, Pauline, Ata, Ezgi, Oguzhan, Anil, Vincent, Yvan, Shaden, Sébastien, Sandra, Guillaume, Bertrand, Elisa, Benjamin, Hector, Jaime (the big one), Thomas, Arsham, Lorenzo (Bastianello), Gürdal, Djamel.

Un mot très spécial pour deux personnes qui me sont très précieuses. Les mots ne suffisent pas pour les remercier. Je remercie du fond du coeur Gül et Yasar pour leur soutien indéfectible depuis mon arrivée en France. Vous avez toujours été à mes côtés à tout moment. "Tout simplement, j'ai grandi avec vous". Je vous assure que cette thèse n'aurait pas vu pas le jour sans vous ! Je remercie également Ayse Yuva qui a toujours été mon idole académique et qui m'a toujours très inspiré. Je remercie très chaleureusement Engin Mete, Nilüfer Mete et Adnan Mete pour ses conseils avisés.

I am thankful to my very close friends in Paris and in Istanbul: Öykü Özbal (my best student), Alara Uçak, İlayda Alyanak, Özer Akman, Enis Günaydin, Enes Karaçayir, Kardelen Eroglu, Eda Saylam, Dicle Durmaz, Aslı Emek, Can Deniz Gürlek, Eser Arisoy, Nursel Arisoy, Sinem Senkur, Emre Özkan, Yasemin Bozdogan, Burcu Özcengiz, Ipek Sahinler, Batuhan Özcan, Yigit Can Par (paradam!), Tara Civelekoglu, Esra Akinci, Maxime Guinnebault, Gilles Grivaud, Selim Kirilmaz, Tunç Durmaz and Serra Ispahani for her support.

Special thanks to NDS team. We are always close friends since 2004!: Hande, Talar, Erdener, Kaan, Serlin, Sabin and Ece.

Every friend is very special but let me thank some of them separately;

Uzay, my "comrade" for our planned revolution in Turkey! We have started our revolution by creating a song list! This is already a good start! Thank you so much for your friendship! You were just near me to help when I was sick two weeks before the submission of the final manuscript of this thesis!

Neslihan, I always enjoyed so much our unique discussions about many interesting topics. I learned so many interesting things about coup d'etats in Turkey from you. I think you are one of the very rarest people with whom I feel very comfortable. Now, it is your time for the Ph.D.!

Çağla, you are the best "panpa" that I have ever had in my life. Especially, I always enjoyed our discussions at the weirdest hours of the day like 3 or 4 a.m. in the morning.

Büke, you are one of my oldest friends (since 1996?). Thanks so much for your friendship and for giving me very delicious cakes when you were in Paris last year.

Ece, my words remain insufficient to express how you are special to me. You are the person who gave me the definition of "a best friend". By writing these lines for you, I am listening to our song "sen benim şarkılarımsın". Thank you so much for your unfailing continuous-time support since our high school years.

I wish to thank all the members of our music band Karlin: Olivier, Clément (Pépé) and Julien who taught me so many things in the music! We composed together some nice piece of music. Thank you, guys!

I owe a special thanks to Harika Balay whom I love as my sister since my childhood. She has been the very first person who made me love the French language that I speak every day!

I would also like to thank Ahmet Enver Çetinçelik. Maybe he has been the only person to listen to my craziest ideas! I always enjoyed his classes when I was in the high school. We became friends thanks to the music and especially to a legendary rock album "Dark Side of the Moon" of Pink Floyd! I do not know how many rock bands and movies I discovered thanks to his broad knowledge of "everything". I am also grateful to Ümran Çetinçelik and Muammer Çetinçelik who always inspired me.

Cansu, I have met many people and discovered some new and interesting ideas during my thesis years. However, you have been my last and best discovery without any doubt! Thank you so much for our sincere discussions about everything at your place. I learned and always learning so much from you. You can not imagine how your presence helped me to smooth the last stressful months of this long thesis journey. This quote that you like so much from Stalker of Tarkovsky is for you: "*...because weakness is a great thing, and strength is nothing...*".

I could not have written this thesis without the unconditional love and care of my parents, Servet and Aysun Mavi. Let me thank them in Turkish. "Sevgili anne, baba, bu tez sizin emekleriniz ve sevginiz olmadan yazılamazdı. Hayatım boyunca en zor anlarda desteğinizi esirgemediğiniz için sonsuz teşekkürler.”

I also thank my aunt Hale, his husband Cengiz and my lovely cousins, Selin, Derin, Asli, Batuhan and Haktan for their love and support. I am thankful to my grandparents Nusrat, Perihan Külah and Aysel Mavi for their constant love, support and care.

Last but not least, I have been told the best real-life stories when I was a young little boy by my grandfather. He has been the very first person who made me think about life, ethics and moral. I dedicate this thesis to his memory...

Can Askan Mavi

Paris, August, 25 2018

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Introduction

1.1 L'adaptation, l'atténuation et les trappes à pauvreté

Dans cette introduction, nous résumons tout d'abord chaque chapitre avec ses détails et faisons une revue de la littérature en vue de mettre en valeur la contribution de chaque chapitre de cette thèse.

Le premier chapitre de cette thèse se focalise sur les mesures d'atténuation et d'adaptation au changement climatique dans une économie faisant face aux catastrophes climatiques. On sait pertinemment qu'à la suite d'événements climatiques dangereux pouvant entraîner des conséquences négatives, un planificateur social devrait envisager des moyens pour éviter les dommages. Une réponse directe doit viser à réduire la probabilité qu'un événement dommageable se produise. Dans de nombreux cas, les activités d'atténuation peuvent réduire le risque d'un changement de régime catastrophique en améliorant la qualité de l'environnement. Les activités d'atténuation visent à améliorer la qualité environnementale qui résulte d'une baisse da la probabilité de catastrophe mais ces activités ne peuvent pas éliminer complètement cette dernière.

La réforestation qui améliore les puits de carbone peut constituer un exemple pour l'activité d'atténuation (IPCC (2007))). Les mesures prises pour réduire les pertes dues à l'événement catastrophique peuvent être considérées comme l'adaptation. La gestion des activités d'adaptation et d'atténuation soulève un arbitrage dynamique intéressant que l'on peut qualifier de "dilemme d'adaptation et d'atténuation" dans la littérature économique (Zemel (2015), Tsur and Zemel (2016a), Crépin et al. (2012)).

Pour élaborer davantage sur ces concepts, nous considérons des exemples concrets. Les améliorations de l'efficacité énergétique, les activités telles que la capture et le stockage du carbone, le reboisement s'attaquent aux causes profondes du changement climatiques en réduisant les émissions de gaz à effet de serre. L'activité d'atténuation peut également être considérée comme un outil permettant d'éviter un événement catastrophique (Martin and Pindyck (2015)). Alors que la mise en place des systèmes de protection contre les inondations, le développement des systèmes d'irrigation visent à réduire les dégâts infligés par un événement catastrophique possible et peut donc être considéré comme une mesure d'adaptation. Dans ce contexte, l'adaptation joue un rôle proactif, ce qui signifie qu'elle n'a aucun effet concret avant l'événement catastrophique (Smit et al. (2000) ; Shalizi and Lecocq (2009)). Dans le cadre de ce chapitre, le problème est de décider d'une combinaison optimale des mesures de réduction des risques et de réduction des dommages à un budget donné.

Dans ce premier chapitre, nous étudions la gestion optimale des ressources naturelles et son lien avec les politiques d'adaptation et d'atténuation dans un modèle de croissance simple avec probabilité d'événement catastrophique en fonction du niveau de qualité de l'environnement. Par exemple, le taux d'arrivée d'événements catastrophiques tels que la sécheresse, les mauvaises récoltes et les inondations est lié à l'exploitation du capital naturel.

1.1.1 Revue de la littérature et contribution

La contribution de ce chapitre est triple: la première est d'analyser les implications de la probabilité d'événement catastrophique sur l'occurrence de la multiplicité des équilibres (i.e trappe environnementale). Nous montrons que l'une des raisons des trappes environnementales est liée à la probabilité d'événement catastrophique. La raison pour laquelle les événements catastrophiques peuvent causer la multiplicité des équilibres est la suivante: lorsqu'une économie est confrontée à une probabilité d'événement catastrophique, un deuxième arbitrage apparaît entre la consommation et la probabilité d'événement catastrophique autre que l'arbitrage intertemporel habituel entre la consommation présente et future. Une économie avec de sérieux problèmes de qualité de l'environnement est supposée être plus impatiente en raison du taux de risque endogène, de sorte que les agents ont tendance à augmenter leur consommation à court terme, ce qui accentue plus encore les problèmes de qualité environnementale. Cet arbitrage entre consommation et événements catastrophiques se traduit par un cycle vicieux de "faible niveau de consommation et de qualité environnementales" à long terme qui peut être défini comme une trappe environnementale (ou encore une trappe à pauvreté).

La possibilité de la multiplicité des équilibres dans les modèles de croissance avec un taux d'escompte endogène a déjà été mentionnée. Cependant, ces travaux n'incluent ni la composante d'incertitude ni les aspects environnementaux. Non seulement nous expliquons les raisons derrière les trappes environnementales, mais nous prouvons également l'existence de la multiplicité des équilibres. La preuve mathématique de l'existence pour les trappes environnementeles nous permet de savoir pour quelles valeurs du dommage catastrophique, l'économie se trouve dans une économie à équilibres multiples.

Notre étude porte également sur une littérature substanielle sur l'exploitation des ressources sous l'incertitude. La prise en compte des événements catastrophiques pour une gestion optimale commence avec Cropper (1976) qui constate que l'épuisement des ressources est plus rapide ou plus lent si le stock de ressources disponible est incertain. Clarke and Reed (1994) proposent un cadre dans lequel le problème dynamique stochastique pour une gestion optimale est transformé en problème déterministe pouvant être résolu avec le principe du maximum de Pontryagin. Les auteurs constatent un effet ambigu du taux de risque sur les émissions de pollution optimales en cas d'événement unique qui réduit indéfiniment l'utilité à un niveau constant. Un travail plus récent de Polasky et al. (2011) présente un modèle de croissance avec un risque d'effondrement exogène / endogène. Ils constatent que lorsque le stock de ressources renouvelables diminue après l'événement catastrophique, l'effet du risque endogène sur l'exploitation des ressources est ambigu. Un article plus récent de de Zeeuw and Zemel (2012) suit une approche de modélisation similaire à celle de Polasky et al. (2011) et tente de déterminer les conditions dans lesquelles l'économie tend à devenir précautionneuse. Ils montrent que l'effet ambigu est dû aux caractéristiques de la fonction valeur après la catastrophe. S'il y a un événement catastrophique et que l'activité économique cesse, le taux de risque entre en dans le taux d'actualisation, ce qui rend la politique de gestion ambiguë. Ren and Polasky (2014) étudient également la gestion optimale sous le risque d'un changement de régime potentiel et montrent qu'il est également possible que l'économie adopte une politique de gestion agressive avec un risque endogène peu élevé, différent de nombreuses études montrant que l'économie devient prudente ou l'effet global est ambigu.

La littérature existante tente de comprendre la réaction de l'économie face à des événements incertains. L'un des points communs des articles susmentionnés est que l'incertitude pousse l'économie à être plus prudente. Cependant, ces études ont négligé la possibilité d'équilibres multiples qui impliquent une réaction hétérogène contre des événements incertains. Quelle est exactement l'hétérogénéité face à l'incertitude ? Une économie peut se trouver dans un point d'équilibre avec une qualité environnementale faible ou élevée. Par conséquent, une économie de haute qualité environnementale est plus prudente que l'économie au niveau de qualité environnementale faible. En d'autres termes, une économie qui se trouve à un équilibre avec une faible qualité environnementale adopte une politique d'exploitation agressive par rapport à une économie à l'équilibre avec un haut niveau de qualité environnementale. En ce sens, notre article montre que l'économie adopte un comportement prudent avec incertitude, mais le niveau de prudence n'est pas le même si une économie converge vers le niveau de qualité environnementale faible ou élevé sur le long terme.

La deuxième contribution principale du chapitre est de présenter un nouvel arbitrage entre adaptation et atténuation autre que l'arbitrage dynamique habituel mis en évidence par de nombreuses études (Bréchet et al. (2012), Kama and Pommeret (2016), Millner and Dietz (2011)). Un travail récent de Zemel (2015) étudie la dynamique transitionnelle de la combinaison optimale d'adaptation et d'atténuation dans un modèle de croissance simple avec incertitude. Tsur and Zemel (2016a) développe Zemel (2015) en enlevant l'hypothèse de linéarité sur les investissements d'adaptation et trouve des solutions intérieures optimales. Cependant, ces études n'ont pas tenu compte de la multiplicité des équilibres et de ses implications en ce qui concerne les politiques d'adaptation et d'atténuation. En effet, notre contribution est de montrer que la politique d'adaptation augmente la possibilité des trappes environnementales. D'autre part, la politique d'atténuation rend plus probable que l'économie admette un équilibre unique. Par conséquent, se concentrer sur les trappes environnementales dans une économie à taux de risque endogène nous permet de présenter un nouvel arbitrage entre adaptation et atténuation pour les trappes environnementales.

La troisième contribution consiste à analyser si les investissements d'adaptation et d'atténuation sont complémentaires ou substituables. Les articles récents de Tsur and Zemel (2016a) et Zemel (2015) négligent si l'adaptation et l'atténuation sont des substituts ou des compléments. Dans ce chapitre, nous montrons que la substituabilité entre ces deux politiques dépend de la façon dont le coût d'adaptation est modélisé. Lorsque les coûts d'investissement liés à l'adaptation sont considérés comme une désutilité sociale comme dans Tsur and Zemel (2016a) et Zemel (2015), l'adaptation et l'atténuation sont toujours complémentaires. Cependant, lorsque le coût de l'adaptation est pris en compte dans la contrainte des ressources, l'adaptation et l'atténuation peuvent également être des substituts si le bénéfice marginal des investissements d'atténuation est suffisamment élevé.

Quel est le rôle de l'adaptation et de l'atténuation sur les trappes environnementales ? Notre contribution est de montrer que la politique d'adaptation peut susiciter des équilibres multiples alors que l'atténuation peut l'éviter. Cela dépend étroitement de la probabilité d'occurrence. La raison de ce résultat est la suivante: on démontre que le capital d'adaptation réduit le niveau optimal stationnaire de qualité de l'environnement puisque les agents s'inquiètent moins des conséquences d'un événement catastrophique avec une capacité d'adaptation croissante. Ensuite, puisque la probabilité d'événement catastrophique augmente, l'arbitrage entre la consommation présente et la probabilité d'événement catastrophique se resserre, ce qui risque de causer des équilibres multiples. Contrairement à ce mécanisme, l'activité d'atténuation améliore la qualité de l'environnement et l'arbitrage entre la consommation présente et la probabilité d'événement catastrophique devient plus faible. Pour mieux comprendre ce résultat, supposons un instant que l'activité d'atténuation puisse éliminer le risque d'événement catastrophique [1]. Ensuite, l'arbitrage entre consommation présente et événement catastrophique disparaît puisque la probabilité d'événement catastrophique disparaît. Par conséquent, la trappe environnementale n'existe pas.

1.2 La croissance schumpétérienne: Adaptation vs Atténuation

Le deuxième chapitre porte sur le risque de catastrophe dans un modèle de croissance schumpétérienne. Nous faisons un pas de plus pour répondre aux questions suivantes: Comment la probabilité d'événement catastrophique affecte-t-elle le processus de destruction créatrice dans l'économie ? Quel est l'effet de la taxe sur la pollution sur le taux de croissance et les implications de la probabilité de catastrophe sur cet effet ? Comment le marché ajuste-t-il le niveau d'équilibre de l'adaptation et de l'atténuation lorsqu'il fait face à une probabilité de catastrophe plus élevée ?

Nombreux rapports récents (voir EU-Innovation (2015)) [2] ont commencé à mettre en évidence l'importance de construire une économie de marché qui gère les services d'adaptation et

[1]Dans notre modèle, il existe toujours un risque d'événement catastrophique quel que soit le niveau de qualité de l'environnement. C'est aussi la justification d'une politique d'adaptation proactive.

[2]La définition des services climatologiques donnée dans ce rapport est la suivante: "Nous attribuons à ce terme un sens large qui couvre la transformation des données relatives au climat informations - produits personnalisés tels que projections, prévisions, informations, tendances, analyses économiques, évaluations (y compris l'évaluation de la technologie), conseils sur les meilleures pratiques, développement et évaluation de solutions et tout autre service en rapport avec le climat la société en général. Ces services comprennent des données, des informations et des connaissances qui favorisent l'adaptation, l'atténuation et les catastrophes."

d'atténuation grâce à des innovations en R&D afin de créer une économie sobre en carbone et résiliente au climat. Le marché des services climatologiques vise à fournir une connaissance en climat à la société par le biais d'outils d'information. Ces services impliquent une analyse très détaillée des connaissances environnementales existantes et des activités de R&D qui informent la société des impacts du climat. De plus, ces services fournissent les informations nécessaires pour prendre des mesures contre les événements extrêmes. En résumé, on peut dire que le but des services climatologiques est de créer un lien entre l'innovation et l'esprit d'entreprise, ce qui pourrait créer de nouvelles opportunités pour la croissance économique.

Ces dernières années, le changement climatique a commencé à être considéré comme une opportunité commerciale pour les entreprises pouvant développer un nouveau service ou produit pour s'adapter aux événements catastrophiques. Ces produits et services devraient assurer la compétitivité pour les entreprises du marché qui favorisent la croissance. En ce qui concerne l'évolution récente des activités d'adaptation et d'atténuation, une analyse de marché décentralisée est plus que nécessaire pour pouvoir analyser rigoureusement les implications de long terme de l'adaptation et de l'atténuation.

Notre objectif dans ce deuxième chapitre est de voir comment l'adaptation et l'atténuation peuvent être possibles grâce à l'activité de R&D gérée par l'économie de marché exposée à un événement catastrophique. À notre connaissance, il n'existe aucune étude traitant des activités d'adaptation et d'atténuation dans un cadre de marché décentralisé, en tenant compte de l'incertitude pour les événements catastrophiques. Notre contribution repose sur la construction d'un modèle de croissance décentralisé qui analyse les politiques d'adaptation et d'atténuation. De plus, les études existantes analysent ces politiques sur des modèles de croissance exogène et le progrès technologique endogène est une composante manquante (voir Zemel (2015), Bréchet et al. (2012), Tsur and Zemel (2016a), Tsur and Withagen (2012), de Zeeuw and Zemel (2012)). En ce sens, notre étude est la première à se concentrer sur l'adaptation et l'atténuation par le biais d'un processus de R&D endogène.

1.2.1 Revue de la littérature et contribution

Pour informer le lecteur sur l'analyse de l'adaptation et de l'atténuation, Bréchet et al. (2012), Kama and Pommeret (2016), Kane and Shogren (2000) and Buob and Stephan (2011) sont les premières études analytiques qui travaillent le "mix" optimal de l'adaptation et de l'atténuation. Cependant, ces études axées sur l'arbitrage entre adaptation et atténuation ne tiennent pas compte de l'incertitude sur les événements climatiques. Pour combler cette lacune dans la littérature, Zemel (2015) and Tsur and Zemel (2016a) introduit l'incertitude de Poisson dans Bréchet et al. (2012) et montre qu'une probabilité plus élevée d'événement catastrophique induit plus de capital d'adaptation à long terme.

Nous revenons maintenant à la littérature sur la croissance schumpétérienne. Aghion and Howitt (1997) sont les toutes premières études combinant l'environnement et les modèles de croissance schumpétériens. Les auteurs introduisent la pollution dans une modèle de croissance schumpétérienne et effectuent une analyse de sentier de croissance équilibrée en tenant compte du critère de développement durable. Grimaud (1999) développe ce modèle et en fait une extension avec une économie décentralisée dans laquelle il propose des instruments environnementaux comme des subventions à la R&D et permis de pollution amenant l'économie à l'optimum.

Hart (2004) est l'une des premières tentatives de modélisation des aspects environnementaux dans un modèle de croissance schumpétérien. Il étudie l'effet d'une taxe sur la pollution et constate que la politique environnementale peut être une politique gagnant-gagnant en diminuant l'intensité de la pollution et en favorisant le taux de croissance à long terme. Dans le même ordre d'idées, Ricci (2007) montre dans un modèle de croissance schumpétérienne que la croissance à long terme de l'économie repose sur l'accumulation des connaissances. Dans son modèle, la réglementation environnementale pousse les producteurs les plus compétitifs à utiliser des biens intermédiaires plus propres. La différence importante entre Hart (2004) et Ricci (2007) est que Ricci (2007) traite un continuum de différents biens intermédiaires (différents en intensité de pollution). Cependant, Hart (2004) propose un modèle dans lequel il n'existe que deux biens intermédiaires. En raison de cette différence de modélisation, Ricci (2007) montre que le renforcement de la politique environnementale ne favorise pas la croissance économique car la contribution marginale de la R&D à la croissance économique diminue. Cependant, l'incertitude sur les événements climatiques est totalement ignorée dans ces modèles. L'un des objectifs de cette étude est d'analyser comment les résultats obtenus par Hart (2004) et Ricci (2007) peuvent changer drastiquement par rapport à une possibilité d'événement catastrophique.

Dans ce chapitre, différent de Hart (2004) et Ricci (2007), le bénéfice de la R&D est double; Tout d'abord, en supposant que les pays les plus riches résistent plus facilement aux événements catastrophiques (voir Mendhelson et al. (2006)), nous montrons que faire de la R&D augmente la richesse de l'économie et la rend plus résistante aux catastrophes. La connaissance ne sert d'outil d'adaptation que si l'événement catastrophique se produit. En ce sens, la connaissance joue également un rôle proactif pour l'adaptation. Deuxièmement, la R&D diminue l'intensité de la pollution des biens intermédiaires (c'est-à-dire l'atténuation) comme dans Ricci (2007) et augmente la productivité totale, ce qui permet un taux de croissance plus élevé sur le sentier de croissance équilibrée.

Dans le cadre de ce chapitre, nous montrons qu'il existe deux effets opposés de la probabilité de catastrophe sur le taux de destruction créatrice. Le premier canal est simple, une probabilité plus élevée d'événement catastrophique augmente le niveau d'impatience des agents. Il s'ensuit que le taux d'intérêt sur le marché a tendance à augmenter. Par conséquent, la valeur espérée d'un brevet de recherche et développement diminue ainsi que la répartition de la main-d'œuvre dans ce secteur. Celui-ci peut être appelé effet d'escompte.

Le second canal est plus intéressant: lorsque la probabilité d'événement catastrophique est plus élevée, le bénéfice marginal de l'activité de R&D augmente puisque le stock de connaissances contribue à renforcer la résilience de l'économie face à la pénalité infligée suite à un événement catastrophique. Par conséquent, le taux d'intérêt sur le marché diminue et la valeur espérée des brevets de R&D augmente. Celui-ci peut être appelé effet d'adaptation.

En d'autres termes, plus le taux de risque augmente, plus le coût d'opportunité de ne pas investir dans la R&D augmente. En résumé, un taux de risque plus élevé peut pousser l'économie à investir davantage dans les activités de R&D. Nous montrons qu'après un certain seuil de pénalité, une augmentation de la probabilité de catastrophe augmente le taux de destruction créatrice dans l'économie. Cela est dû au fait que l'effet d'adaptation domine l'effet d'escompte.

Nos résultats indiquent que le niveau d'adaptation du marché par rapport au niveau d'atténuation dépend du rapport entre l'intensité de la pollution et le taux de productivité total. Plus le secteur de la R&D offre davantage de biens intermédiaires plus propres, moins l'économie s'adapte aux dégâts climatiques. Cela repose sur l'hypothèse habituelle selon laquelle les produits intermédiaires plus propres sont moins productifs. Ensuite, avec des biens intermédiaires plus propres, le taux de croissance et l'accumulation de connaissances sont moins élevés. En effet, l'arbitrage entre adaptation et atténuation (voir Bréchet et al. (2012)) n'est présent dans notre modèle vu que l'adaptation et l'atténuation viennent de la même source qui est le secteur de R&D. Étant donné que l'adaptation et l'atténuation se développent à un rythme de croissance équilibré, l'économie augmente à la fois l'adaptation et l'atténuation à chaque date. L'élément intéressant est qu'il existe un nouvel arbitrage entre l'adaptation et la pollution qui pourrait apparaître dans l'économie. L'activité de R&D diminue l'intensité de la pollution mais, parallèlement, elle cherche à augmenter la productivité totale de l'économie qui augmenterait l'échelle de l'économie. Si l'effet d'échelle domine la baisse de l'intensité des émissions, le taux de croissance augmente. Cependant, dans ce cas, la croissance de la pollution est plus élevée, même avec des biens intermédiaires plus propres, étant donné l'échelle de l'économie. Ceci est proche de ce que l'on appelle le paradoxe de Jevons, qui affirme que les améliorations technologiques augmentent l'efficacité énergétique mais entraînent une pollution plus élevée à long terme.

Il convient de noter que les entreprises diminuent leur intensité de pollution car elles sont confrontées à une taxe sur la pollution prélevée sur l'utilisation de biens intermédiaires polluants. Ils font donc de la R&D pour réduire l'intensité de la pollution afin de réduire la charge fiscale. Notre modèle montre un effet positif de la taxe de pollution sur la croissance, comme dans Ricci (2007), la baisse de la demande de biens intermédiaires entraînant un déplacement de la main-d'œuvre vers le secteur de la R&D. Nous montrons qu'un taux de risque plus élevé peut accroître l'effet positif de la pression fiscale verte sur le taux de croissance de l'économie à long terme, si le taux de pénalité est suffisamment élevé. Cet effet est dû à un avantage marginal plus élevé de la R&D car il aide une économie à mieux réagir aux catastrophes.

1.3 Les préférences indvidiuelles: Cycles Limites

Le troisième chapitre de la thèse se concentre sur la question autour de la soutenabilité dans une économie confrontée à une possibilité d'événement catastrophique, à travers une analyse de cycle limite. Le fait que des événements climatiques catastrophiques incertains puissent causer des dommages à grande échelle est largement reconnu Alley et al. (2003), Stocker et al. (2012)). Un nombre considérable d'études se concentre sur la prise de décision concernant la politique d'exploitation des ressources naturelles en situation d'incertitude (Bretschger and Vinogradova (2017), Tsur and Zemel (1998, 2016c); **?**, Clarke and Reed (1994)). De plus, certaines publications récentes mettent l'accent sur la politique environnementale optimale pour faire face à l'incertitude. À cette fin, les politiques d'adaptation et d'atténuation et leurs implications sous l'incertitude sont l'un des principaux points d'intérêt des travaux récents (Zemel (2015), Tsur and Zemel (2016a), Mavi (2017)).

1.3.1 Revue de la littérature et contribution

A part la littérature sur l'incertitude et l'exploitation des ressources, une autre partie de la littérature se concentre sur la relation entre actualisation et durabilité qui suscite des débats intenses dans la littérature économique. En particulier, le débat s'est intensifié dans le contexte du changement climatique (Stern (2006),Weitzman (2007), Heal (2009)).

Certaines des études portent sur le rôle des préférences temporelles individuelles (voir Endress et al. (2014), Schneider et al. (2012), Marini and Scaramozzino (1995, 2008), Burton (1993)). La présence de préférences temporelles individuelles dans un modèle économique est intéressante parce que le modèle d'agent à durée de vie infinie est critiqué pour ne pas respecter les préférences individuelles du consommateur.

Les articles cités ci-dessus qui intègrent le taux d'actualisation individuel dans leurs modèles sont basés sur le cadre proposé par Calvo and Obstfeld (1988). Les auteurs introduisent des préférences de temps individuelles (c'est-à-dire un taux d'actualisation individuel) dans un modèle à générations imbriquées. Ensuite, ils trouvent statiquement le niveau de consommation globale de toutes les générations à un moment donné. Une fois l'agrégation effectuée, le modèle se réduit à un modèle à agent représentatif. Ce cadre a été utilisé en économie de l'environnement pour traiter divers sujets importants tels que l'équité entre générations. Cependant, ces articles n'analysent pas le rôle des préférences temporelles individuelles sur la dynamique globale à long terme. Cela introduit clairement une dichotomie entre un modèle à générations imbriquées et le modèle d'agent à durée de vie infinie, car on ne connaît pas les implications du taux d'actualisation individuel pour la dynamique à long terme dans ce modèle d'agent à durée de vie infinie. L'un des objectifs de ce chapitre est de répondre à ce besoin et d'analyser les implications du taux d'actualisation individuel (ou du taux d'actualisation familial) sur la dynamique agrégée en profondeur.

Cependant, d'une part, les études traitant les impacts à long terme de l'incertitude sur la politique d'exploitation des ressources ne prennent pas en compte la durabilité et l'équité intergénérationnelle (voir Bommier et al. (2015), Zemel (2015)). D'autre part, le volet de la littérature sur la durabilité et l'équité intergénérationnelle ne prend pas en compte les événements incertains (voir Burton (1993); Endress et al. (2014); Marini and Scaramozzino (1995)). En ce sens, nous pouvons affirmer que le lien entre la durabilité et les événements catastrophiques est négligé dans la littérature en économie de l'environnement et que ce chapitre vise à combler cette lacune importante.

Dans ce troisième chapitre, nous avons deux motivations importantes: premièrement, nous visons à montrer l'importance des préférences individuelles pour la durabilité lorsque l'économie est exposée à des cycles limites (bifurcation de Hopf). Deuxièmement, nous montrons que les cycles limites à long terme sont optimaux mais non conformes au critère de Développement Durable. Ensuite, nous défondons l'idée que le critère de développement durable devrait être révisé pour inclure également les cycles limites.

La contribution de ce chapitre est double: premièrement, en développant le cadre Calvo and Obstfeld (1988) pour tenir compte des événements incertains, nous montrons que pour certains paramètres critiques pour le taux d'actualisation individuel (familial), les cycles endogènes (bifurcation de Hopf) peuvent survenir dans l'économie à long terme. Le mécanisme derrière les cycles limites peut être résumé comme suit: d'une part, l'économie accumule du capital physique et crée des déchets. En ce sens, l'environnement est utilisé comme ≪puits≫ par l'économie. Cela peut être considéré comme un objectif économique. D'autre part, comme les dégâts infligés après l'événement catastrophique sont proportionnels au stock de ressources naturelles restant après l'événement, l'économie souhaite protéger le capital naturel. C'est l'objectif environnemental. Lorsqu'il devient difficile de décider entre ces deux politiques conflictuelles, il peut être préférable de faire des cycles autour de l'état stationnaire [3]. Enfin, certaines valeurs de paramètres pour le taux d'actualisation individuel peuvent rendre plus ou moins difficile le pilotage entre l'objectif économique et l'environnement.

Afin de mieux comprendre la motivation du chapitre et la raison pour laquelle nous utilisons un modèle à générations imbriquées. Voici quelques précisions importantes : en fait, l'existence des cycles limites est possible même sans utiliser un modèle à générations imbriquées. On peut facilement montrer que les cycles limites ont aussi lieu dans un modèle d'agent représentatif (voir Wirl (2004)). En d'autres termes, la principale source des bifurcations est l'arbitrage susmentionné et non la structure de la population. Cependant, cela ne veut pas dire que le taux d'actualisation individuel n'a pas d'importance pour les bifurcations. Le taux d'actualisation individuel est important dans la mesure où il peut

[3]Notez que les cycles autour de l'état stationnaire sont optimaux.

être plus ou moins difficile de choisir entre les objectifs économiques et environnementaux. Pour certains niveaux du taux d'actualisation individuel, il devient difficile de décider entre l'objectif environnemental et économique. C'est cette difficulté qui fait apparaître les cycles. Par conséquent, nous trouvons important de se concentrer sur le taux d'escompte individuel dans cette étude.

On peut aussi dire que cet arbitrage entre l'objectif économique et environnemental est habituel dans les modèles de croissance comportant des aspects environnementaux. L'occurrence des cycles limites peut également être comprise grâce à la complémentarité des préférences (voir Dockner and Feichtinger (1991), Heal and Ryder (1973)). À cette fin, nous montrons rigoureusement que, sans les déchets découlant de l'accumulation de capital physique et la probabilité d'événement catastrophique, les préférences de l'économie au fil du temps sont indépendantes au sens de Koopmans (voir Koopmans (1960)). Il s'ensuit que l'économie admet un équilibre stable à long terme.

Comment la complémentarité des préférences au fil du temps peut-elle expliquer les cycles limites ? S'il y a une augmentation incrémentale de la consommation proche de la date t_1, cela implique une réallocation de la consommation entre les dates futures t_2 et t_3 (par exemple, une partie de la consommation passe de t_2 à t_3) s'il y a complémentarité des préférences dans le temps, il s'ensuit que la consommation augmente à la date t_1 et t_3 et diminue à la date t_2. Si la complémentarité des préférences est suffisamment forte, ces fluctuations se produiront en boucle pour toujours.

En effet, les cycles limites sont largement étudiés en économie de l'environnement. Wirl (1999, 2004) and Bosi and Desmarchelier (2016, 2017) étudient l'existence des cycles limites dans des modèles avec un cadre d'agent représentatif. Néanmoins, aucune de ces études ne lie les cycles limites à l'équité entre les générations et à la durabilité au sens du critère du développement durable.

À ce stade, la question à se poser est la suivante: quelles sont les implications des cycles limites en matière de durabilité ? Notez que le critère de développement durable exige que l'utilité de la consommation ait un chemin non décroissant (c.-à-d. $\frac{du(c(t))}{dt} \geq 0$. Si l'économie est exposée à des cycles limités en raison de l'arbitrage entre l'objectif environnemental et économique et / ou la complémentarité des préférences, le critère du développement durable n'est pas respecté puisque l'utilité et la dynamique des stocks de ressources naturelles sont cycliques. comportement à long terme.

Deuxièmement, contrairement au cadre Calvo and Obstfeld (1988) et aux articles utilisant ce cadre, nous montrons que les préférences temporelles individuelles peuvent modifier les propriétés de stabilité du modèle. Ce résultat réfute le résultat conventionnel selon lequel la dynamique des agrégats est uniquement régie par le taux d'actualisation du planificateur social (voir Endress et al. (2014), Schneider et al. (2012), Marini and Scaramozzino (1995, 2008), Burton (1993)). En effet, nous montrons que le taux d'actualisation individuel joue un rôle important dans la durabilité d'une économie.

La première partie du modèle étant un modèle OLG, il existe une répartition intra-générationnelle de la consommation qui est stable dans le temps. Nous montrons également que l'équité intra-générationnelle peut être conforme à la durabilité puisque nous montrons qu'une répartition plus égale de la consommation entre les générations garantit un équilibre stable à long terme.

Quel est le lien entre le taux d'escompte individuel et les cycles limites ? La raison pour laquelle le taux d'actualisation individuel peut augmenter les cycles limites est le suivant: lorsque le planificateur social agrège la consommation sur tous les agents, il trouve une fonction d'utilité agrégée et une fonction post-valeur dépendant du taux d'actualisation social et individuel. Différents niveaux de taux d'actualisation individuels modifient le poids de la fonction d'utilité et la fonction de post-valeur dans la fonction objective du planificateur social. Cela équivaut à dire que le compromis entre l'objectif économique et l'objectif environnemental change en fonction du taux d'actualisation individuel différent.

On soutient l'idée que la durabilité et l'équité intergénérationnelle sont généralement perçues comme des enquêtes normatives (Solow (2005, 2006)). Ensuite, un planificateur social qui prête attention à la durabilité et à l'équité intergénérationnelle devrait chercher à éviter les cycles limites. Nous montrons que le planificateur social peut éviter les cycles limites par une politique environnementale visant à protéger l'environnement. Cela est dû au fait qu'un stock de ressources naturelles plus élevé implique une utilité marginale plus faible de la consommation. En conséquence, les différents niveaux du taux d'actualisation individuel ne devraient pas modifier beaucoup l'arbitrage entre l'objectif économique et l'objectif environnemental. Par conséquent, il y a moins de chance qu'une économie soit exposée aux cycles limites.

Chapter 2

Catastrophic Events, Adaptation, Mitigation and Environmental Traps

Can Askan Mavi

Doubt is an uncomfortable condition, but certainty is a ridiculous one[1].

2.1 Introduction

A social planner should consider ways of avoiding damage as a result of hazardous events that might entail negative consequences. A direct response requires action to reduce the probability of a harmful event taking place. In many cases, mitigation activities are able to reduce the risk of a catastrophic regime shift by improving the environmental quality[2] altough they can not eliminate it completely. In such situations, a possible action could be alleviating the negative consequences of a catastrophic damage. The measures taken

[1]The quote belongs to Voltaire and it is written in a letter addressed to Frederick William, Prince of Prussia, 28 November 1770.

[2]Since we focus on environmental quality in the chapter, mitigation activities are aimed at increasing environmental quality. A possible definition of mitigation activity can be reforestation, which enhances carbon sinks. (IPCC (2007))

to reduce the loss due to the catastrophic event can be considered as adaptation. The management of adaptation and mitigation activities raises an interesting dynamic trade-off that can be described as "adaptation and mitigation dilemma" in the environmental economics literature (Zemel (2015), Tsur and Zemel (2016a), Crépin et al. (2012)).

To elaborate more on these concepts, we consider concrete examples. Improvements in energy efficiency, activities such as carbon capture and storage and reforestation address the root causes, by decreasing greenhouse gas emissions and reducing the risk of a catastrophic climate event and therefore can be referred to as mitigation. Mitigation activity can also be seen as a tool to avert a catastrophic event (Martin and Pindyck (2015)). Whereas, installing flood defenses and developing irrigation systems aim to reduce the damage inflicted by a possible catastrophic event and hence can be classified as adaptation. In this context, adaptation plays a *proactive* role, which means that it has no concrete effect prior to the catastrophic event (Smit et al. (2000) ; Shalizi and Lecocq (2009)). In this example, the problem is to decide an optimal combination of risk-reducing and damage-reducing measures within a given budget.

In this chapter, we study the optimal management of natural resources and its link with adaptation and mitigation policies in a simple growth model under catastrophic event probability depending on the environmental quality level[3]. For example, the arrival rate of catastrophic events such as droughts, crop failures and floods [4] is linked to the exploitation of natural capital. Our model uses a general definition for natural capital which encompasses all natural amenities such as the stock of clean water, soil and air quality, forests, biomass and so on.

The contribution of this chapter is threefold: The first contribution of the chapter is to analyse the implications of the catastrophic event probability on the occurrence of the multiplicity of equilibria (i.e the environmental trap[5]). We show that one of the reasons behind environmental traps is the catastrophic event probability. The reason why catastrophic events may cause a poverty trap is as follows: when an economy faces a catastrophic event probability, a second trade-off arises between consumption and the catastrophic event proba-

[3] We use interchangeably the terms environmental quality and natural resource stock.

[4] The depletion of forests in a region increases the probability of floods, since the soil loses its ability to absorb the rainfall.

[5] We use the term poverty trap and environmental trap interchangeably.

bility other than the usual intertemporal trade-off between present and future consumption. An economy with serious environmental quality problems is supposed to be more impatient due to the endogenous hazard rate. Therefore, agents tend to increase their consumption at earlier dates since they face a higher event probability, which again stresses the environmental quality over time. This trade-off between consumption and catastrophic events results in a vicious cycle of "low level of environmental quality and consumption" in the long run that can be defined as a poverty trap.

The possibility of multiple stationary equilibria in growth models with endogenous hazard[6] is already mentioned by Tsur and Zemel (2016c). However, one cannot understand the economic intuition behind the multiplicity of equilibria Tsur and Zemel (2016c). In this chapter, not only do we offer an economic explanation for environmental traps, but we also prove the existence of the multiplicity of equilibria. The mathematical proof of the existence of environmental traps allows us to know within which range of catastrophic damage, the economy finds itself in a multiple equilibria economy.

Our study relates also to a substantial literature on resource exploitation under uncertainty. The consideration of catastrophic events for optimal management starts with Cropper (1976), who finds that the depletion of resources is either faster or slower if the available resource stock is uncertain. Clarke and Reed (1994) offer a framework where the stochastic dynamic problem for optimal management is transformed into a deterministic problem that can be solved with the Pontryagin maximum principle. The authors find an ambiguous effect of hazard rate on the optimal pollution emissions when there is a single occurrence event that reduces indefinitely the utility to constant level. A more recent work by Polasky et al. (2011) presents a growth model with exogenous/endogenous collapse risk. They find that the renewable resource stock decreases after the event and the effect of the endogenous risk is ambiguous. A paper by de Zeeuw and Zemel (2012) follows a similar modeling approach to Polasky et al. (2011) and tries to figure out the conditions under which the economy tends to become precautious. They show that the ambiguous effect is due to the characteristics of the post-event value. If there is a doomsday event and economic activity stops, the hazard rate enters as an additional factor to the discount rate, which makes the

[6]There is also a large literature on growth models with endogenous discounting pointing out the possibility of multiple equilibria (Das (2003), Drugeon (1996), Schumacher (2009)). However, these papers do not include neither the uncertainty component or the environmental aspects.

management policy ambiguous with respect to catastrophic risk. Ren and Polasky (2014) also study optimal management under the risk of potential regime shift and show that it is also possible that the economy adopts an aggressive management policy under a small endogenous event risk, different from many studies which show that the economy becomes either precautious or the overall effect is ambiguous.

The existing literature tries to figure out the reaction of the economy to uncertain events. One of the common arguments in the above-cited papers is that the uncertainty pushes the economy to be more precautious. However, these studies overlooked the possibility of multiple equilibria which can imply a heterogeneous reaction to uncertain events. What exactly is the heterogeneity against the uncertainty? An economy can find itself at an equilibrium point with low or high environmental quality. Therefore, an economy at high environmental quality is more precautious than one at low environmental quality level. In other words, an economy at the low environmental quality equilibrium adopts an aggressive exploitation policy relative to an economy at the high environmental quality equilibrium. In this sense, this chapter shows that, given uncertainty, an economy adopts precautionary behaviour, although the level of precaution is not the same when an economy converges to a low or a high environmental quality level.

The second and the main contribution of the chapter is to present a new trade-off between adaptation and mitigation other than the usual dynamic trade-off highlighted by numerous studies (Bréchet et al. (2012), Kama and Pommeret (2016), Millner and Dietz (2011)). A recent work by Zemel (2015) studies the time profile of the optimal mix of adaptation and mitigation in a simple growth model with uncertainty. Tsur and Zemel (2016a) extend Zemel (2015) by relaxing the linearity assumption on adaptation investments and find optimal interior solutions. However, these studies did not take into account the multiplicity of equilibria and its implications regarding the adaptation and mitigation policies. Indeed, our contribution is to show that adaptation policy increases the possibility of poverty traps. On the other hand, mitigation policy makes more likely that the economy admits a unique equilibrium. Hence, concentrating on poverty traps in an economy with endogenous hazard rate allows us to present a new trade-off between adaptation and mitigation regarding the poverty traps.

The third contribution is to analyze whether adaptation and mitigation investments are complementary or substitutes. The recent papers by Tsur and Zemel (2016a) and Zemel (2015) overlooks whether adaptation and mitigation are substitutes or complementary. In this chapter, we show that substitutability depends on how the adaptation cost is modeled. When adaptation investment costs are considered as a social disutility, as in Tsur and Zemel (2016a) and Zemel (2015), adaptation and mitigation are always complementary. However, when the adaptation cost is taken in the resource constraint[7], adaptation and mitigation can also be substitutes if the marginal benefit from mitigation investments is sufficiently high.

What is the role of adaptation and mitigation on the poverty traps mentioned above? Our contribution is to show that adaptation policy can cause multiple equilibria while mitigation can avoid it. This depends largely on the occurrence probability. This is because adaptation capital is shown to decrease the optimal steady state level of environmental quality, since agents worry less about the consequences of a catastrophic event when there is an increasing adaptation capacity. Then, since the catastrophic event probability increases, the trade-off between present consumption and the catastrophic event probability becomes tighter or more difficult, which is likely to raise multiple equilibria. Contrary to this mechanism, mitigation activity improves the environmental quality and the trade-off between present consumption and catastrophic event probability turns to be easier. To better understand this result, assume for a moment that mitigation activity can eliminate the catastrophic event risk[8]. Then, the trade-off between present consumption and hazardous event disappears, since the catastrophic event probability disappears. Consequently, the poverty trap is not a possible outcome.

The remainder of the chapter is organized as follows: Section 4.2 presents the benchmark model. Section 2.3 describes the model with adaptation and mitigation policies and the section 2.4 explains in a greater-depth the implications of adaptation and mitigation on the occurrence of poverty traps. Section 2.5 provides numerical illustrations and Section 2.6 concludes the chapter.

[7]This means that adaptation cost is financed by the rents stemming from the natural capital.

[8]In our model, a risk of a catastrophic event exists always whatever the environmental quality level is. This is also the justification for a proactive adaptation policy.

2.2 Model

Let $S(t)$ represent environmental stock available or environmental quality at time t, e.g, the stock of clean water, soil quality, air quality, forests, biomass. We refer to a broad definition of environmental quality which encompasses all environmental amenities and existing natural capital that have an economic value[9]. Obviously, disamenities such as waste and pollution stemming from consumption decrease environmental quality stock. The stock $S(t)$ evolves in time according to

$$\dot{S}(t) = R(S(t)) - c(t) \tag{2.1}$$

where the control variable $c(t)$ stands for consumption at time t. With a given initial state $S(0)$, an exploitation policy of environmental stock $c(t)$ generates the state process $S(t)$ according to the equation (2.1) and provides the utility $u(c(t), S(t))$. Similar to Kama and Schubert (2007), we use a framework where consumption comes directly from environmental services and causes environmental damages.

We make use of the following assumptions.

A.1 The regeneration of environmental quality is characterised by $R(.) : \mathbb{R}_+ \to \mathbb{R}_+$, $R(S) > 0$ and $R''(S) < 0$.

A.2 The utility function $u(.) : \mathbb{R}_+ \to \mathbb{R}_+$ is twice continuously differentiable with the following properties ; $u(c) < 0$, $u'(c) > 0$, $u''(c) < 0$, $\forall c$ and $\lim_{c\to 0} u'(c) = \infty$.

An important issue highlighted by Schumacher (2011) is related to the use of a utility function with a positive domain in growth models with endogenous discount. Schumacher (2011) shows rigorously that the models with endogenous discounting should use a utility function with a positive domain since the utility function shows up in the optimal path of consumption. Indeed, since our model focuses on catastrophic recurrent events, the utility function does not show up in the Keynes-Ramsey rule. Therefore, the use of a utility function with a negative or positive domain does not lead to qualitatively different solutions.

[9] We exclude mining and oil industry from our definition of natural capital.

In addition to the fact that the environmental stock $S(t)$ represents a source of consumption for the economy, it also affects the occurrence probability of a catastrophic event. To elaborate this in greater depth, forests, for example influence considerably the environmental conditions in a given area. (Dasgupta and Mäler (1997), chapter 1) and helps to decrease the probability of catastrophic events (see Jie-Sheng et al. (2014), Bradshaw et al. (2007))[10]. When catastrophic events occur, they inflict environmental damages. The consequences of these recurrent catastrophic events are defined by the post-event value $\varphi(S)$ that we will discuss later.

Let T be the event occurrence time and let $F(t) = Pr\{T \leq t\}$ and $f(t) = F^{'}(t)$ denote the corresponding probability distribution and density functions, respectively. The environmental stock dependent hazard rate $h(S)$ is related to $F(t)$ and $f(t)$ with respect to

$$h(S(t))\Delta = Pr\{T \in (t, t+\Delta \mid T > t)\} = \frac{f(t)\Delta}{1 - F(t)} \tag{2.2}$$

where Δ is an infinitesimal time interval. We have $h(S(t)) = -\frac{ln(1-F(t))}{dt}$. The term $h(S(t))\Delta$ specifies the conditional probability that a catastrophic event will occur between $[t, t+\Delta]$, given that the event has not occurred by time t. A formal specification for probability distribution and density functions gives

$$F(t) = 1 - exp\left(-\int_0^t h(S(\tau))\,d\tau\right) \;\; and \;\; f(t) = h(S(t))\,[1 - F(t)] \tag{2.3}$$

Since $S(t)$ is a beneficial state, the hazard rate h is a non-increasing function (a higher environmental quality stock entails a lower occurrence probability, $h^{'} < 0$). Given the uncertain arrival time T, the exploitation policy $c(t)$ yields the following payoff

$$\int_0^T u(c(t))\,e^{-\rho t}dt + \varphi(S(T))\,e^{-\rho T} \tag{2.4}$$

[10] An interesting real world example could be the reforestation project in Samboja Lestari conducted by Borneo Orangutan Survival Foundation. The project helped to increase rainfall by 25% and to avoid droughts by lowering air temperature by 3 degrees Celcius to 5 degrees Celcius . (Boer (2010), Normile (2009))

where ρ is the social discount rate. We consider recurrent events which entail an immediate damages $\bar{\psi}$. Recurrent events are repeated events such as droughts and floods that occur frequently in our current society. The post-value function describing the economy with recurrent events after the occurrence of catastrophic event is defined as

$$\varphi(S) = V(S) - \bar{\psi} \tag{2.5}$$

Taking expectations of the expression (2.4) according to distribution of T and considering (2.3) gives the expected payoff

$$V(S) = \max_{c(t)} \int_0^\infty \left[u(c(t)) + h(S(t))\,\varphi(S(t))\right] \exp\left(-\int_0^t \left[\rho + h(S(\tau))\right] d\tau\right) dt \tag{2.6}$$

The solution of maximising (2.6) with respect to evolution of environmental stock (2.1) leads to the Keynes-Ramsey rule (see Appendix (2.A) for details). For notational convenience, we drop the time index t,

$$\dot{c} = -\frac{u_c(c)}{u_{cc}(c)}\left[R_S(S) - \rho - \frac{\bar{\psi} h_S(S)}{u_c(c)}\right] \tag{2.7}$$

When there are recurrent catastrophic events, the economy adopts a precautionary behavior due to the presence of the term $\frac{\bar{\psi} h_S(S)}{u_c(c)}$ in equation 2.7. This result is similar to de Zeeuw and Zemel (2012), where the authors show that recurrent events induce a precautionary behavior.

Proposition 1. *(i) A multiple steady state (i.e a poverty trap) is possible if the economy is exposed to endogenous hazard, depending on the environmental stock S. If the catastrophic event probability is exogenous, the multiple equilibria are not a possible outcome.*

(ii) The necessary conditions to have a poverty trap for recurrent events and single occurrence are given by $\exists S < \bar{S}$[11] such that ;

$$G_S(S) = R_{SS}(S) - \frac{\bar{\psi} h''(S)}{u_c(R(S))} + \frac{\bar{\psi} h_S(S)\, u_{cc}(R(S))\, R_S(S)}{\left(u_c(R(S))\right)^2} > 0 \tag{2.8}$$

[11] $\bar{S}$ is the carrying capacity of the natural resource stock.

Proof. See Appendix (2.B)

In a standard neoclassical growth model, this condition cannot be satisfied, since all terms with endogenous catastrophic event probability vanish and the condition reduces to $R^{''}(S) > 0$, which is not possible according to A.1. In a Ramsey-Cass-Koopmans model, the usual inter-temporal trade-off is between the present and future consumption. Moreover, an economy exposed to catastrophic events faces an additional trade-off between present consumption and catastrophic risk. Indeed, this is the reason why the economy could find itself in a trapped equilibrium.

Before explaining in greater depth the economic intuition behind the occurrence of poverty traps, more concretely, one can understand the occurrence of poverty traps due to hazard rate by making a phase diagram analysis. Recall that the consumption rule is $\dot{c}/c = \sigma(r-\rho)$[12] without catastrophic event probability. Then, the steady-state curve $\dot{c}=0$ is vertical and implies a unique equilibrium, which is not the case with hazard probability. Considering equation (2.7), we can observe that steady state curve $\dot{c}=0$ is non-linear on a phase plane (S,c). Therefore, multiple equilibria is a possible outcome with event risk. To illustrate this explanation, we refer to a phase diagram analysis in the following section.

2.2.1 Phase Diagram Analysis

Finding directions of arrows on the phase diagram analysis requires some focus, since steady state curve of consumption is a function of environmental quality S. If (c,S) is below (above) the $\dot{S}=0$, $R(S)-c > (<)\, 0$, which makes $\dot{S} > (<)\, 0$. The analysis is not that easy for $\dot{c}=0$. Therefore, we use the necessary condition (2.8). Above the $\dot{c}=0$ line, we have $\dot{c} > (<)\, 0$ if $G^{'}(S) > (<)\, 0$. We define different zones on the phase diagram where the slope of $G(S)$ changes. The possible phase diagrams of the dynamical system are as follows :

[12]where $r = R^{'}(S)$. With usual growth model, one can maximise $\int_0^\infty \frac{c^{1-\sigma}}{1-\sigma}e^{-\rho t}dt$ with respect to $\dot{S} = R(S)-c$ and find the usual Euler equation.

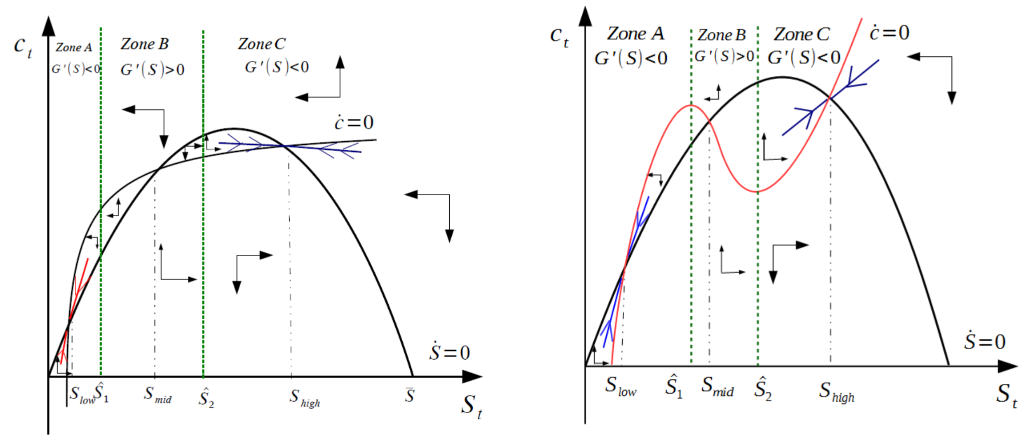


Figure 2.1: Phase diagram with monotonically and non-monotonically increasing $\dot{c} = 0$ curve

Note that, from multiple steady state condition, between $\left[0;\hat{S}_1\right]$ and $\left[\hat{S}_2;\bar{S}\right]$, we know that $G'(S) < 0$ (see Figure (2.B.1) in Appendix *(2.B)*.). Within the zones A and C, the slope of the $\dot{c} = 0$ line is always positive, which means that the curve is always increasing between these intervals (see Appendix *(2.B)* for details). Conversely, within the zone B, it is not possible to determine exactly whether the slope of $\dot{c} = 0$ line is increasing or decreasing. Consequently, we present two different phase diagrams.

The direction of arrows in Figure (2.1) shows that there exist three steady-state, with one being unstable and two others being stable. Furthermore, subsequent analysis of stability of dynamical system stipulates that there could be complex dynamics around the middle steady-state.

Lemma 1. *The steady-states* (S_{low}, c_{low}) *and* (S_{high}, c_{high}) *are saddle path stable. However,* (S_{mid}, c_{mid}) *could have complex dynamics.*

Proof. See Appendix (2.D)

This means that the economy could converge to either high or low equilibrium in terms of environmental quality. Once the economy reaches equilibrium (low or high), it definitely stays there. The economy reaching the low equilibrium is said to be "trapped", where the consumption and environmental quality are lower relative to high equilibrium.

How can a country be trapped to low equilibrium? The economic explanation is as follows: postponing consumption is too costly for survival ($u^{'}(0) = \infty$) for poor agents and preferences are directed toward the present. In this case, agents are exploiting most of the natural capital at earlier dates and they face a higher event probability. Since the occurrence probability is high, agents tend to be impatient[13] and start to excessively exploit natural resources. Consequently, a vicious cycle occurs due to the trade-off between present consumption (i.e use of environment) and the hazard rate.

According to our theoretical model, the hazard rate pushes the economy to be precautionary (see the Keynes-Ramsey rule (2.7).). However, developed countries (high equilibrium) become more conservative about the environment relative to developing countries (low equilibrium) when there is an endogenous hazard rate. Indeed, Environmental Performance Index in 2016[14] supports this result, since many African and Asian countries are listed at the bottom of the rankings. A striking real world example can be the deforestation trend in Asian countries and rainforest loss in African countries. Margano et al. (2014) show that there was a loss of 40% of the total national forests in Indonesia between the period 2000-2012.

Low environmental quality and the overuse of environmental assets[15] represent important environmental concerns in many African and Asian countries (Environmental Outlook to 2030, OECD (2008)). Our theoretical model shows that hazardous event probability may be an important factor in understanding why some countries are trapped at lower environmental quality and consumption levels. One may say that the occurrence probability can trigger catastrophic events due to the vicious cycle effect and cause poverty traps.

With all these elements, one may understand how environmental conditions through catastrophic event probability could cause a poverty trap in a country. Indeed, since exploitation of the environment is one of the major sources of revenue in developing countries[16], countries with a low initial state of environmental quality are likely to suffer from a poverty trap due

[13]The adjusted discount rate is $\rho + h(S)$.

[14]The Environmental Performance Index is a method developed by Yale Center for Environmental Law and Policy, that evaluates environmental policies of countries. see `http://epi.yale.edu/country-rankings`.

[15]Some examples could be permanent, such as clean water stress, decreased soil quality and lack of clean water stocks.

[16]see `http://data.worldbank.org/indicator/NY.GDP.FRST.RT.ZS?name_desc=false&view=map` for a detailed data on forests rents.

to the mechanism explained above.

2.2.2 An example of multiple equilibria

Note that multiple equilibria are an issue related to functional forms and parameter values. For this reason, we do not limit ourselves to giving necessary and sufficient conditions for the existence of multiple equilibria but we also give conditions on the constant exogenous parameters by the use of common functional forms used in the literature. The regeneration of the environment, hazard rate and the utility function respectively are given as follows :

$$R(S) = g(1-S)S \tag{2.9}$$

$$h(S) = \left(1-\bar{h}S^2\right) \tag{2.10}$$

$$u(c) = log(c) \tag{2.11}$$

where g is the intrinsic growth rate of the environmental quality and $0 < \bar{h} < 1$ shows at which extent the catastrophic event probability depends on the environmental quality level. At the steady-state, the equation (2.7) can be written as

$$G(S) = R_S(S) - \rho - \frac{\bar{\psi}h_S(S)}{u_c(c)} = 0 \tag{2.12}$$

By using the functional specifications, we reformulate the equation (2.12) combined with (2.1) at the steady state

$$-2g\bar{\psi}\bar{h}S^3 + 2g\bar{\psi}\bar{h}S^2 - 2gS + (g-\rho) = 0 \tag{2.13}$$

Proposition 2. *The sufficient condition to have three positive real roots for the third degree polynomial equation (2.13) is as follows:*

$$\begin{aligned}&\frac{\left(4g^2+36g(g-\rho)-27(g-\rho)^2\right)-\sqrt{\left(4g^2+36g(g-\rho)-27(g-\rho)^2\right)^2-2^9g^3(g-\rho)}}{8(g-\rho)2g\bar{h}}\\ &<\bar{\psi}<\frac{\left(4g^2+36g(g-\rho)-27(g-\rho)^2\right)+\sqrt{\left(4g^2+36g(g-\rho)-27(g-\rho)^2\right)^2-2^9g^3(g-\rho)}}{8(g-\rho)2g\bar{h}}\end{aligned} \tag{2.14}$$

Proof. See Appendix (2.E)

This result is not only a mathematical condition but also suggests that the damage rate due to catastrophic events should not be either too low or too high. When the damage rate is too high, this implies that the economy adopts a precautionary behavior since the marginal benefit of protecting the environment is higher. On the other hand, when the damage rate is too low, the economy tends to care less about the environment and exploits natural resources. However, when the damage is between a certain range of values, the economy is neither very precautionary nor careless about the environment. This also can explain the presence of multiple equilibria in the economy. When the damage is neither very high nor very low, the economy is trapped in environmental traps (see Wirl (2004)). In case where the damage is $\bar{\psi} = 0$, the trade-off between present consumption and catastrophic events vanishes, since the catastrophic event probability does not change the optimal consumption behaviour.

2.3 Model with Environmental Policy

In this section, we consider the benchmark model and analyse how adaptation and mitigation policies affect the main results obtained in the model without environmental policy. In particular, our focus will be on the implications of adaptation and mitigation policies for poverty traps.

As mentioned in the benchmark model, when a catastrophic event occurs, the economy suffers from environmental damage. However, a social planner could reduce the damage ψ via adaptation capital K_A. This kind of modeling adaptation along the same lines as Zemel (2015) and Tsur and Zemel (2016c) differs from Bréchet et al. (2012) and Kama and Pommeret (2016), where the adaptation capital affects directly the damage function for all time t. In our specification, as mentioned above, adaptation plays a *proactive* role. Hence, tangible benefits of adaptation can be gained only if a catastrophic event occurs. However, this is not to say that investing in the adaptation decision makes no difference. Its contribution is accounted for by the objective function of the social planner. Investing at rate A contributes to adaptation capital K_A, which follows the stock dynamics

$$\dot{K}_A(t) = A(t) - \delta K_A(t) \tag{2.15}$$

where δ represents the capital depreciation rate and damage function $\psi(K_A)$ decreases when adaptation capital K_A increases. We assume that

A.3 The damage function is characterised by $\psi(.)$: $\mathbb{R}_+ \to \mathbb{R}_+$, $\psi(0) = \bar{\psi}$, $\psi(\infty) = \underline{\psi}$, $\psi(K_A) > 0$, $\psi'(K_A) < 0$ and $\psi''(K_A) > 0$

When there is no adaptation expenditure, the inflicted damage will be a constant term as in Tsur and Zemel (2016c). Moreover, it is realistic to assume that reduction in damage has a limit, since we cannot get rid of the negative effects of a catastrophic event completely by accumulating adaptation capital. For that reason, we assume that damage function is constrained between an upper and a lower bound.

Investing at rate M for mitigation that improves the environmental quality. Then, in the presence of mitigation activity, environmental quality evolves according to

$$\dot{S}(t) = R(S(t)) + \Gamma(M(t)) - c(t) \tag{2.16}$$

where $\Gamma(M)$ represents the effects of mitigation that encompasses all activities such as reforestation, desalination of water stock, enhancing carbon sinks, etc. Mitigation is defined as a "human intervention to reduce the sources or enhance the sinks of greenhouse gases." (IPCC-Report (2014), p.4) In this sense, reforestation can be considered as a means to enhance carbon sinks since forests allow for carbon sequestration.

The specification of the mitigation variable is similar to that in Chimeli and Braden (2005). Alternatively, function $\Gamma(M)$ can be considered as "environmental protection function". The expenditures for environmental protection may be directed not only toward pollution mitigation but also toward protection of forests and recovery of degraded areas. Equivalently, mitigation activity can be seen as a means of improving environmental quality.

In order to keep the model as simple as possible, we choose to consider mitigation as a flow variable. We use the following assumption

A.4 The mitigation function given by $\Gamma(.) : \mathbb{R}_+ \to \mathbb{R}_+$ is twice continuously differentiable, with the following properties ; $\Gamma'(M) > 0$, $\Gamma''(M) < 0$.

The mitigation function is assumed to be an increasing and concave function. Note that mitigation activity can be considered as a complement to the regeneration of environment. In an economy with adaptation and mitigation activities, the expected payoff is

$$\int_0^\infty \left[u(c(t)) - Q^1(A) - Q^2(M) + h(S(t))\varphi(S(t))\right] \exp\left(-\int_0^t [\rho + h(S(\tau))]\, d\tau\right) dt \tag{2.17}$$

$Q^1(A)$ and $Q^2(M)$ are the cost functions for adaptation and mitigation respectively that enter in the utility function as a social cost as in Zemel (2015) and Tsur and Zemel (2016c). In order to simplify the calculations, we assume a linear cost function for mitigation: $Q^2(M) = P_M M$ where P_M is the unit price for mitigation, and a convex cost function for adaptation activity: $Q^1(A) = \phi_A \frac{A^2}{2}$. The reason behind the choice of a convex cost function for adaptation activity instead of a linear function is that the accumulation of adaptation capital is linear. Otherwise, a linear cost function for adaptation investments leads to corner solutions (see Zemel (2015) for an example). Since we assume a concave function for mitigation activity $\Gamma(M)$, the use of a linear cost function gives optimal interior solutions and simplifies calculations.

The optimal policy is to maximise (2.17) subject to (2.16), (2.15) and to the additional state variable that comes from the endogenous hazard rate (see details in Appendix (2.F)). Essentially, the control variables $c(.)$, $M(.)$ and $A(.)$ are determined by shadow prices $\lambda(.)$ and $\mu(.)$ corresponding to S and K_A.[17].

Since the focus of our study is to figure out the effect of adaptation and mitigation policies on poverty traps, we need to know how the steady-state level of adaptation capital K_A and mitigation activity M change with respect to environmental quality stock S. More precisely, the objective is to understand the effect of these two central policies on necessary condition (2.8) for multiple equilibria.

[17]Another possible and probably easier way to present our results would be to use the L-method of Tsur and Zemel (2016a) for multi-state control problems. However, this method allows an analysis only on long term properties and not on transitional dynamics. Since we conduct a numerical analysis to justify the main mechanisms of the model, we choose to use standard optimal control methods to solve the problem at hand.

A steady-state of the optimisation program (2.17) is a couple (K_A, S) approached by an optimal process $(K_A(.), S(.))$. Once the process reaches steady-state, the hazard rate $h(S)$ becomes constant and behaves as another component of the discount factor. As stated in Zemel (2015), the problem at hand is a deterministic one and the objective function is also deterministic, yielding a value that corresponds to the maximum expected value of the uncertainty problem. If the process enters a stationary state, it would stay at point (K_A, S) indefinitely, without being disturbed.

Occurrence of the catastrophic event causes a damage, but otherwise, the social planner keeps the same optimal steady state policy prior to occurrence. This is not to say that inflicted damage does not have any effect on decision making of the social planner. The negative consequences of a possible catastrophic event are already taken into account in the objective function (2.17).

Considering the loci for possible steady states in the (K_A, S) plane (see equation (A1.71) in Appendix (2.J)), we have

$$Q_A^1(\delta K_A)(\rho+\delta) = -\psi_{K_A}(K_A)\, h(S) \tag{2.18}$$

which defines the function $S(K_A)$. The graph of $S(.)$ represents a curve in the (K_A, S) plane, denoting the steady-state curve and having the following economic intuition : right-hand side can be interpreted as the marginal benefit of adaptation capital and left-hand side is the marginal cost of adaptation capital. Optimal steady states are located on this curve, which takes the slope

$$\frac{dS}{dK_A} = \frac{(\rho+\delta)\, Q_A^1(\delta K_A) + \psi_{K_A K_A}(K_A)\, h(S)}{h_S(S)\, \psi_{K_A}(K_A)} < 0 \tag{2.19}$$

The negative slope indicates that when the environmental quality is higher, the economy needs less adaptation capital. This is plausible, since the probability of the catastrophic event is lower. Correspondingly, one may say that when the economy accumulates more adaptation capital, natural resources start to be overused.[18] The trade-off between adap-

[18] Zemel (2015) also finds a similar result.

tation and mitigation is evident in (2.19). Making mitigation increases the environmental quality. It follows that occurrence hazard decreases, which case pushes the economy to accumulate less adaptation capital. We can also explain this result by looking at the catastrophic risk factor in (2.17). When environmental quality increases, the weight of this component decreases due to lower hazard rate, which means that there is less incentive to accumulate adaptation capital.

Another interpretation of this result can be as follows: since agents expect to face less damage with an adaptation policy and can more easily bear the negative consequences of a catastrophic event, they tend to care less about the environmental quality.

Following the same type of analysis in order to see the relationship between environmental quality and mitigation activity, we use the first-order condition $u_c\left(R\left(S\right)+\Gamma\left(M\right)\right)\Gamma_M\left(M\right)=Q_M^2\left(M\right)=P_M$ (see equations (A1.54) and (A1.55) in Appendix (2.H)). To facilitate calculations, we suppose a linear cost function for mitigation activity: $Q_2\left(M\right)=P_M M$, where P_M is the unit price for mitigation. Optimal steady states should satisfy (A1.54) and (A1.55). To see how the steady-state level of mitigation activity M changes with respect to environmental quality S, we calculate

$$\frac{dS}{dM}=-\frac{u_{cc}\left(c\right)\left(\Gamma_M\left(M\right)\right)^2+u_c\left(c\right)\Gamma_{MM}\left(M\right)}{u_{cc}\left(c\right)R_S\left(S\right)\Gamma_M\left(M\right)}<0 \tag{2.20}$$

The equation (2.20) will be important to analyse the effect of mitigation activity on the occurrence of poverty traps. It follows that when environmental quality is higher, the social planner undertakes mitigation. In other words, the economy needs less mitigation if environmental quality is higher. To sum up, from (2.19) and (2.20), one may observe that when environmental quality is higher, the economy chooses to invest less in adaptation and mitigation.

2.3.1 Adaptation and mitigation : complementarity vs substitutability

An important issue regarding the adaptation and mitigation policies is to know whether adaptation and mitigation are complementary or substitute policies (chapter 18, IPCC

(2007)). We solve the model with both adaptation and mitigation policy and simply calculate (see Appendix 2.F for calculations)

$$\frac{dK_A}{dM} = -\frac{\underbrace{h_S\left(.\right) R_S^{-1}\left(.\right)\left(u_{cc}^{-1}\left(.\right)\left(-\frac{P_M}{\left(\Gamma_M\left(M\right)\right)^2}\right) - \Gamma_M\left(M\right)\right)\frac{\psi_{K_A}\left(K_A\right)}{Q_A^1\left(\delta K_A\right)}}_{<0}}{\underbrace{\frac{h\left(S\right)\psi_{K_AK_A}\left(K_A\right)}{Q_A^1\left(\delta K_A\right)} - \frac{h\left(S\right)\psi_{K_A}\left(K_A\right)Q_{AA}\left(\delta K_A\right)\delta}{\left(Q_A^1\left(\delta K_A\right)\right)^2}}_{>0}} > 0 \tag{2.21}$$

The equation (2.21) states that when the economy increases its mitigation activities, the adaptation investments increase as well if $R_S\left(S\right) > 0$. Then, adaptation and mitigation activity are complementary. This result is important for environmental traps. If the economy increases both adaptation and mitigation, a policymaker should be aware of the possibility of a poverty trap caused by an excessive level of adaptation relative to mitigation activity.

However, note that this result is sensitive to how one models the adaptation and mitigation cost in the dynamic optimisation program. If the adaptation cost enters in the resource constraint instead of a disutility, adaptation and mitigation can also be substitutes. We find

$$\frac{dK_A}{dM} = -\frac{\underbrace{h_S\left(.\right) R_S^{-1}\left(.\right)\left(u_{cc}^{-1}\left(.\right)\left(-\frac{P_M}{\left(\Gamma_M\left(M\right)\right)^2}\right) - \Gamma_M\left(M\right)\right)\frac{\psi_{K_A}\left(K_A\right)\Gamma_M\left(M\right)}{Q_A^2\left(\delta K_A\right)P_M}}_{<0} + \underbrace{\frac{h\left(.\right)\psi_{K_A}\left(K_A\right)\Gamma_{MM}\left(M\right)}{P_M Q^2\left(A\right)}}_{>0}}{\underbrace{\frac{h\left(.\right)\psi_{K_AK_A}\left(K_A\right)}{Q_A^2\left(\delta K_A\right)} - \frac{h\left(.\right)\psi_{K_A}\left(K_A\right)Q_{AA}\left(\delta K_A\right)\delta}{\left(Q_A^2\left(\delta K_A\right)\right)^2} + h_S\left(.\right) R_S^{-1}\left(.\right)\psi_{K_A}\left(K_A\right)\delta}_{>0}} \gtreqless 0 \tag{2.22}$$

The implications of modeling the cost of adaptation are neglected in the existing literature, which takes adaptation cost as a disutility (Tsur and Zemel (2016a); Zemel (2015)). This example clearly shows that adaptation and mitigation can be either complementary or substitutes depending on how one models the adaptation cost. What is the economic intuition behind this result? Since adaptation investments are now financed by the rents stemming from natural resources, the economy may decrease its adaptation capital when investing in mitigation activity which aims at increasing the natural capital. Analysing the equation (2.22), one may easily observe that the additional term $\frac{h(.)\psi_{K_A}(K_A)\Gamma_{MM}(M)}{P_M Q(A)}$ in the nominator relates to the marginal benefits of mitigation activity. When the marginal benefit of mitigation is sufficiently high, the economy can only invest in mitigation to avert the catastrophic event risk. Indeed, since adaptation is financed by natural resources, it contributes in the increase of the catastrophic event risk. Therefore, the economy prefers to avert the risk rather than adapting to a catastrophic damage. In a sense, we can also argue that investing less in adaptation is equivalent to mitigating more when adaptation is financed by natural resources.

If the marginal benefit of mitigation is not sufficiently high, the economy cannot decrease the catastrophic event probability. Then, the adaptation capital becomes necessary in order to deal with a catastrophic event. In a such case, adaptation and mitigation activities are expected to be complementary.

In the numerical analysis section, we also verify that adaptation can always cause multiple equilibria when the cost function for the adaptation show up in the resource constraint.

2.4 Can environmental policy cause/avoid a poverty trap?

In this section, we highlight the mechanisms to understand under what conditions environmental policy could cause or avoid a poverty trap. To facilitate understanding of the mechanisms, we prefer to analyse separately the effect of adaptation and mitigation policies on poverty traps.

2.4.1 An economy implementing only adaptation policy

We argue that adaptation policy alone might trap an economy in a low equilibrium. The mechanism is as follows: when a policymaker starts to invest in adaptation capital, the environmental quality decreases as shown in (2.19) and the hazard rate increases. Since the preferences of low-income countries are directed towards the present[19], an increase in hazard rate due to adaptation capital accumulation amplifies the impatience of low-income countries. Then, these countries become aggressive in exploiting more natural resources. It follows that the event risk amplifies again, with an increasing need for adaptation capital[20]. This mechanism, yielding a vicious cycle, explains why a developing country with a high level of marginal utility may get trapped into a low steady-state equilibrium by investing only in adaptation capital. In order to assess the effect of adaptation capital on the poverty trap, we calculate the necessary condition for multiple equilibria (see Appendix (2.J) for details)

$$G_S(S) = R_{SS}(S) - \frac{h_{SS}(S)\,\psi(K_A)}{u_c(R(S))} + \frac{h_S(S)\,\psi(K_A)\,u_{cc}(R(S))\,R_S(S)}{(u_c(R(S)))^2} \underbrace{- \left[\frac{\psi_{K_A}(K_A)\,h_S(S)}{u_c(R(S))}\right]\frac{dK_A}{dS}}_{=Z_1>0} > 0 \tag{2.23}$$

The term Z_1 stands for the effect of adaptation capital on the poverty trap. Since the term Z_1 is negative, we can say that the possibility of multiple equilibria increases with adaptation capital. The trade-off between present consumption and a catastrophic event becomes more significant with adaptation policy, since the catastrophic event risk increases.

2.4.2 An example with only adaptation policy

To prove the existence of the multiple equilibria in an economy with adaptation policy, we start with an example of an economy without any environmental policy that admits a

[19]Recall from equation (2.7) that the multiple equilibria occur when agents are poor and not willing to postpone their consumption. Then, implementing adaptation policy increases the hazard rate.

[20]Since the catastrophic event risk increases, the marginal value of adaptation capital increases.

unique equilibrium and use the following usual functional specifications.

$$R(S) = gS(1-S) \tag{2.24}$$

$$h(S) = \left(1-\bar{h}S\right) \tag{2.25}$$

$$u(c) = log(c) \tag{2.26}$$

$$\tag{2.27}$$

Proposition 3. *(i) An economy without an environmental policy admits a unique steady-state.*

(ii) When the economy invests in adaptation activity, multiplicity of equilibria occurs if the following condition on catastrophic damage $\bar{\psi}$ holds

$$\frac{\left[\left(4g^2+36g(g-\rho)-27(g-\rho)^2\right)-\sqrt{\left(4g^2+36g(g-\rho)-27(g-\rho)^2\right)^2-2^9g^3(g-\rho)}\right]}{8(g-\rho)g\left(\bar{h}\right)^2}$$
$$<\bar{\psi}<\frac{\left[\left(4g^2+36g(g-\rho)-27(g-\rho)^2\right)+\sqrt{\left(4g^2+36g(g-\rho)-27(g-\rho)^2\right)^2-2^9g^3(g-\rho)}\right]}{8(g-\rho)g\left(\bar{h}\right)^2} \tag{2.28}$$

Proof. See Appendix (2.E)

This example clearly shows that an economy which is not exposed to multiple equilibria is trapped in an environmental traps by investing in adaptation capital if the condition ($A1.52$) holds.

Consider a benchmark case where the economy without environmental policy admits a unique equilibrium. Then, a social planner takes the benchmark economy and starts to invest in adaptation capital. As stated above, the possibility of a poverty trap increases.

We can remark that with adaptation policy, when the economy suffers from a poverty trap, the high steady-state level of environmental quality is lower than the benchmark unique steady-state level. This means that adaptation policy decreases the steady-state level of environmental quality even for wealthier countries.

Indeed, a policy recommendation based on more adaptation capital in developing countries could trap these countries at lower equilibrium. This is not to say that the social planner should stop investing in adaptation capital. Rather, should be aware of the possible adverse effects of adaptation policy and should implement mitigation policy to avoid the negative effects of adaptation policy.

2.4.3 An economy with mitigation policy

We show that an economy implementing mitigation policy could escape a poverty trap. The reason is as follows: since the consumption comes from environmental assets, improving environmental quality (mitigation) increases the consumption level in the long run. Therefore, low income countries could have a lower level of marginal utility of consumption, implying that they could start to make far-sighted decisions. As catastrophic event probability decreases with mitigation policy, agents will be more patient and willing to postpone their consumption to the future. In order to see this mechanism in a formal way, we provide the necessary condition for multiple equilibria (Appendix 2.H for details)

$$G_S(S) = R_{SS}(S) - \frac{\bar{\psi} h_{SS}(S)}{u_c(c)} + \frac{\bar{\psi} h_S(S) u_{cc} R_S(S)}{(u_c(c))^2} + \underbrace{\frac{\bar{\psi} h_S(S) u_{cc} R_S(S)}{(u_c(c))^2} \frac{dM}{dS}}_{=Z_2<0} > 0 \quad (2.29)$$

Mitigation activity makes it less likely that the conditions necessary for multiple equilibria hold since Z_2 is a negative term. (Appendix (2.H) for details.) In our model, mitigation activity can decrease the probability of a catastrophic event but cannot totally eliminate it. Indeed, this provides a justification to invest in a proactive adaptation capital.

However, suppose for a moment that mitigation activity could reduce the hazard rate to zero. It follows that the trade-off between present consumption and a catastrophic event, which causes multiple equilibria, disappears completely. Therefore, multiple equilibria are not a possible outcome. Based on these elements, one can understand that mitigation activity leads to weakening the trade-off between present consumption and a catastrophic event.

Obviously, mitigation activity not only removes the economy from poverty trap but also increases the steady-state level of environmental quality as expected. Indeed, mitigation policy allows the economy to have a higher consumption level. Hence, low-income countries could postpone their consumption to the future as they can fulfill more easily the basic needs for survival. In a nutshell, a mitigation policy could break the vicious cycle that can be triggered by an adaptation policy. Then, one can conclude that social planner should couple an adaptation policy with a mitigation policy in order to avoid a potential poverty trap.

2.4.4 An example with only mitigation policy

To prove that mitigation can rid the economy of multiple equilibria, we start with an example of a benchmark economy with multiple equilibria and use the following usual functional specifications. In order to have analytical results, we use the following linear functional forms;

$$R(S) = gS(1-S) \tag{2.30}$$

$$h(S) = \left(1 - \bar{h}S^2\right) \tag{2.31}$$

$$u(c) = log(c) \tag{2.32}$$

$$\mathit{\Gamma}(M) = M \tag{2.33}$$

$$Q^2(M) = P_M \frac{M^2}{2} \tag{2.34}$$

Another simplification only used for this example in order to ease the calculations is that the cost function appears in dynamic equation of environmental quality $\dot{S}$.

$$\dot{S} = R(S) + \Gamma(M) - Q^2(M) - c \tag{2.35}$$

For the sake of analytical tractability, we use a linear mitigation function and convex cost function for mitigation. We relax these simplifications in the numerical analysis to show that our result is robust, with more realistic functional forms. The function $G(S)$ for the economy with mitigation is as follows (see Appendix 2.I for Proposition 4)

$$G(S) = -2g\bar{\psi}\bar{h}S^3 + 2g\bar{\psi}\bar{h}S^2 + \left(\frac{\bar{\psi}\bar{h}}{P_M} - 2gS\right) + (g - \rho) = 0 \tag{2.36}$$

Proposition 4. *An economy with mitigation policy is less likely to face multiple equilibria.*

Proof. *See Appendix (2.E)*

2.5 Numerical analysis

This section further illustrates the theoretical findings we obtained for the examples with functional forms that we presented in the previous sections.

2.5.1 The economy with mitigation

First, we start from a benchmark economy without policy that admits multiple equilibria. When the benchmark economy starts to invest in mitigation activity, multiplicity of equilibria is avoided. The equation (2.13) for the benchmark economy and equation (2.36) give the following results[21];

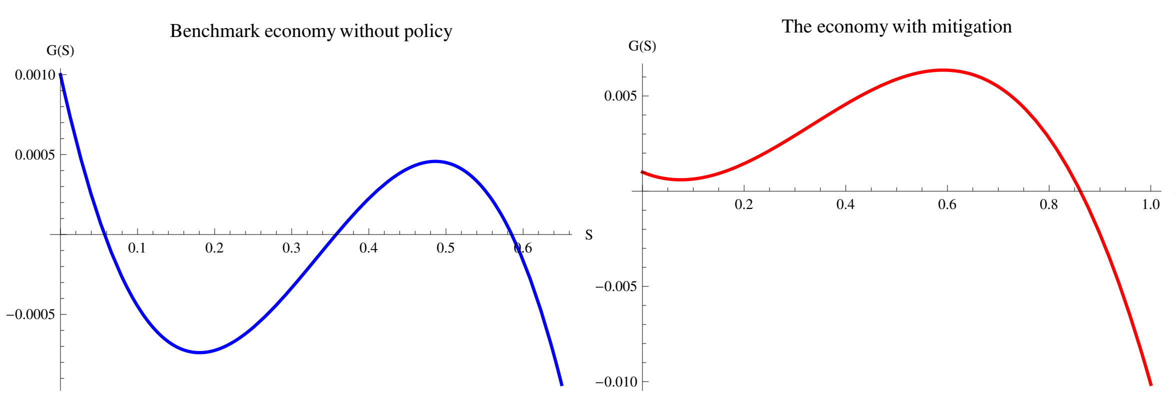


Figure 2.2: The benchmark economy vs the economy with adaptation

[21]We use the following parameter values: $\rho = 0.01$, $g = 0.011$, $\bar{h} = 0.02$, $\overline{\psi} = 190$, $P_M = 350$.

2.5.2 The economy with adaptation

In order to understand the implications of adaptation policy regarding the multiplicity of equilibrium, we start from a benchmark economy that admits a unique equilibrium. From the equation (A1.48), we know that the economy admits a unique equilibrium. When the benchmark economy only invests in adaptation, the multiplicity of equilibria occurs if the condition (A1.14) holds[22].

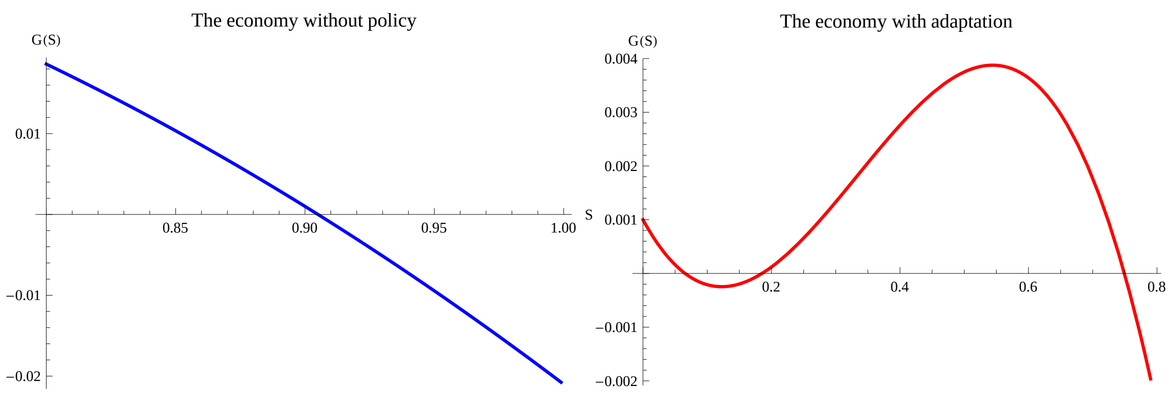


Figure 2.3: The benchmark economy vs the economy with adaptation

2.5.3 A numerical example

In this subsection, we show the same results regarding the effects of adaptation and mitigation on the occurrence of multiple equilibria by relaxing the linearity assumptions on functional forms that we imposed for the analytical tractability. We use the following functional specifications

[22]We use the following parameter values: $\rho = 0.01$, $g = 0.011$, $\bar{h} = 0.5$, $\overline{\psi} = 40$, $\delta = 0, 1$, $\phi = 0, 1$

Natural Regeneration Function : $R(S) = gS\left(1 - \frac{S}{\bar{S}}\right)$
Source : Ren and Polasky (2014)
$\bar{S}$ Carrying capacity of environment g Intrinsic growth rate of the resource stock
Penalty function : $\psi(K_A) = \bar{\psi}\left(\omega + (1-\omega)e^{-\gamma K_A}\right)$
Source : Bréchet et al. (2012)
$\bar{\psi}$ Penalty rate without adaptation policy ω Lower bound of penalty when $\psi(\infty)$ γ Elasticity of adaptation w.r.t to penalty rate
Mitigation function : $\Gamma(M) = M^{\alpha}$
Source : Kama and Pommeret (2016)
α Elasticity of mitigation activity

Utility function : $u(c) = \frac{c^{1-\sigma} - c_{min}^{1-\sigma}}{1-\sigma}$
Source : Bommier et al. (2015)
c_{min} Post-value consumption σ Degree of relative risk aversion
Hazard Function : $h(S) = \frac{2\bar{h}}{1+exp[\eta(s/s^{*}-1)]}$
Source : Ren and Polasky (2014)
$\bar{h}$ Upper bound for hazard rate η Endogeneity level of catastrophic event s^{*} Risk-free steady state of resource stock
Cost of adaptation investment : $Q_1(A) = \phi_A \frac{A^2}{2}$
Source : Quadratic cost function
ϕ_A Parameter for the change of marginal cost of adaptation

We present the benchmark model without policy and the economy with only mitigation policy[23]:

[23]We use the following parameter values for the benchmark model: $\rho = 0.025$, $c_{min} = 1$, $\sigma = 2.75$,$g = 0.05$,$\bar{S} = 10$, $\eta = 8.5$,$\bar{h} = 0.5$, $\overline{\psi} = 500$, $\delta = 0.065$. For the model with mitigation policy, we have two additional parameters: $P_M = 12$, $\alpha = 0.5$.

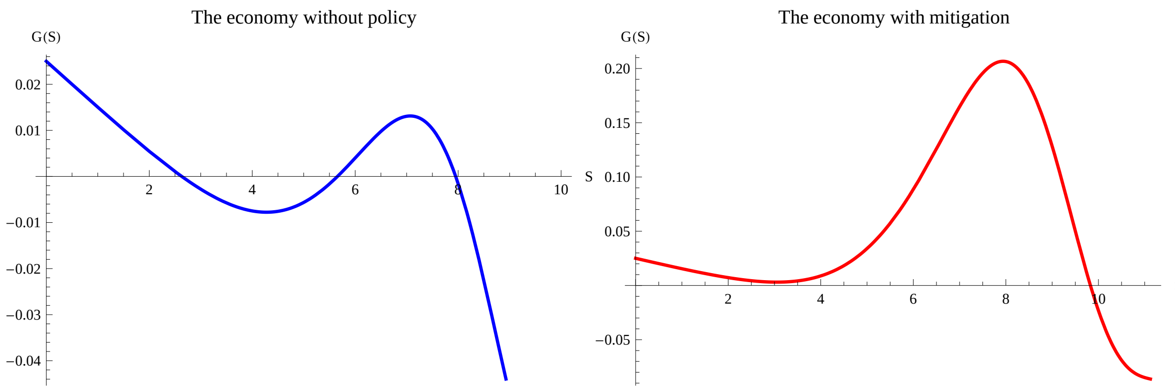


Figure 2.4: The benchmark economy vs the economy with only mitigation

We present the benchmark model without policy and the economy with only adaptation policy[24]:

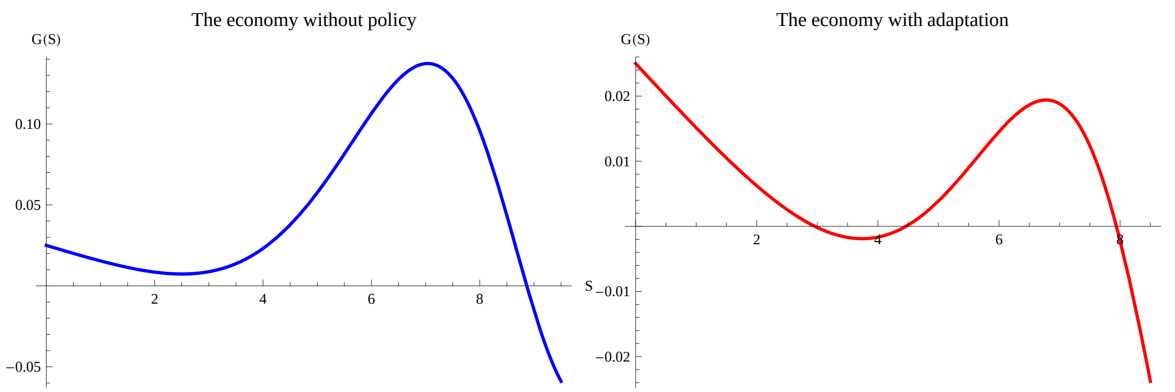


Figure 2.5: The benchmark economy vs the economy with only adaptation

We show that the possibility of multiple equilibria is even possible when the adaptation cost shows up in the resource constraint.

[24] We use the following parameter values for the benchmark model: $\rho = 0.025$, $c_{min} = 1$, $\sigma = 2.5$,$g = 0.05$,$\bar{S} = 10$, $\eta = 7$, $\bar{h} = 0.5$, $\overline{\psi} = 500$,$\delta = 0.065$. For the model with adaptation policy, we have the following additional parameters: $\omega = 0.35$, $\phi = 0.05$, $\gamma = 0.6$.

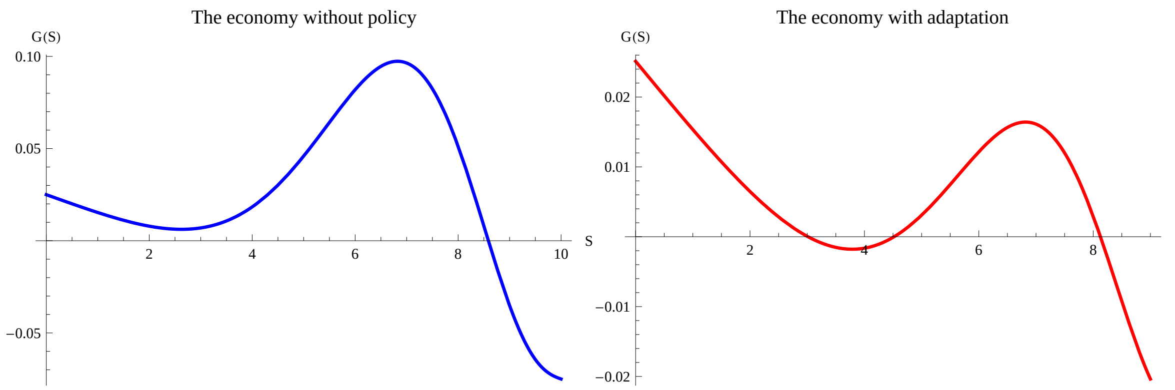


Figure 2.6: The benchmark economy vs the economy with only adaptation

2.6 Conclusion

In this chapter, we analysed the effect of adaptation and mitigation policies on poverty traps in an economy subject to catastrophic event risk. The contribution of the study is to offer a new explanation for poverty traps by the catastrophic event probability and understand how environmental policy plays an important role in causing or avoiding development traps. We believe that this new perspective also provides interesting pointers to policymakers regarding the opposite effects of adaptation and mitigation policies. Our main results show that adaptation policy can lead the economy into a poverty trap, whereas mitigation helps to avoid a poverty trap. We show that a new trade-off appears between adaptation and mitigation with respect to their effect on poverty traps, other than the trade-off between adaptation and mitigation over time mentioned in numerous studies (see Zemel (2015), Tsur and Zemel (2016a), Bréchet et al. (2012)). The fact that adaptation policy could cause a poverty trap does not mean that social planner should not invest in adaptation activity. On the contrary, since it is impossible to eliminate completely the hazardous event risk, she should invest in adaptation capital but should couple this policy with mitigation activity to avoid the adverse effects of adaptation policy. This is because mitigation activity weakens the trade-off between present consumption and catastrophic events, by improving the environmental quality.

Future research could test the model using empirical methods. Currently, this is very challenging since there are no available data on adaptation investments, but it is very desirable and also part of our future research agenda.

Appendix

2.A Derivation of (2.7).

To solve the maximization problem, we write down the Hamilton-Jacobi-Bellman equation.

$$\rho V^B(S) = \max_c \left\{ u(c) + V_S^B(S)(R(S) - c) - h(S)\left(V^B(S) - \varphi(S)\right) \right\} \tag{A1.1}$$

where $V^B(S)$ is the value of the maximization program before the event. As also stated in the text, the value of the problem after the event is as follows:

$$\varphi(S) = V^B(S) - \bar{\psi} \tag{A1.2}$$

The economy is exposed to an inflicted damage after the event. The first-order condition is given by

$$u'(c) = V_S^B(S) \tag{A1.3}$$

Computing the derivative of (A1.42) with respect to the environmental quality stock $S(t)$ yields

$$\rho V_S^B(S) = -\bar{\psi} h_S(S) + V_{SS}^B(S)(R(S) - c) + V_S^B(S) R_S(S) \tag{A1.4}$$

Differentiating the equation (A1.3) and using equations (A1.3) and (A1.4) gives

$$\frac{u_{cc}(c)}{u_c(c)} \dot{c} = \frac{V_{SS}^B(S)}{V_S^B(S)} \dot{S} \tag{A1.5}$$

$$\rho = -\frac{\bar{\psi} h_S(S)}{V_S^B(S)} + \frac{V_{SS}^B(S)}{V_S^B(S)} \dot{S} + R_S(S) \tag{A1.6}$$

Arranging equations (A1.5) and (A1.6) gives the Keynes-Ramsey rule

$$\dot{c} = -\frac{u_c(c)}{u_{cc}(c)} \left[R_S(S) - \rho - \frac{\bar{\psi} h_S(S)}{u_c(c)} \right]$$

2.B Proof of Proposition 1

The first part of the proof starts with an analysis of the limits of a function (let this function be $G(S)$) that describes the steady state of the economy by a single equation in the long run. Then, in the second part, we focus on the form of $G(S)$ function and related necessary conditions for multiple steady state.

(a) In a sense, the function $G(S)$ can be considered as the equation $\dot{c} = 0$ in terms of S at the steady state equilibrium. Writing down equations $\dot{c} = 0$ and $\dot{S} = 0$;

$$R_S(S) - \rho - \frac{\bar{\psi} h_S(S)}{u_c(c)} = 0 \tag{A1.7}$$

$$R(S) - c = 0 \tag{A1.8}$$

Plugging equation (A1.8) in (A1.7) yields

$$G(S) = R_S(S) - \rho - \frac{\bar{\psi} h_S(S)}{u_c(R(S))} \tag{A1.9}$$

Note that we limit our analysis between $S \in [0, \bar{S}]$. It is possible to say that the function $G(S)$ starts with a positive value and starts to have negative when S approaches $\bar{S}$. In this framework, $\bar{S}$ is the level of environmental quality level where, the consumption level is equal zero. With these information, it is easy to verify $\underset{S\to 0}{lim} G(S) = \infty$ and $\underset{S\to \bar{S}}{lim} G(S) = z < 0$.

(b) In this part, we show the necessary conditions for the existence of multiple steady state, which also allows to represent the function $G(S)$;

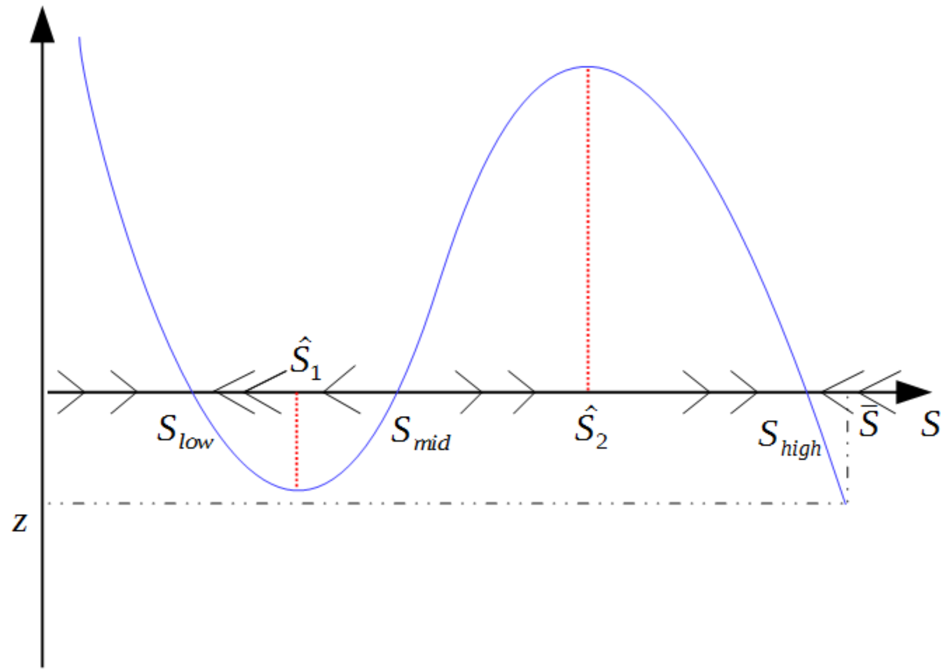


Figure 2.B.1: $G(S)$ function with uncertainty

The sufficient condition for the existence of S_{low} is that $\exists S < \widetilde{S}$ [25] which ensures (i) $G_S(S) > 0$ and (ii) $G(S) < 0$. Unless the condition (i) $G_S(S) > 0$ is satisfied, the function $G(S)$, starting from m may cross x-axis just one more time and converge to z, which results in a unique steady state equilibrium. Additionally, the condition (ii) $G(S) < 0$ is also necessary to ensure that $G(S)$ function crosses the x-axis by S_{low} at least once.

Recall that $\hat{S}_1$ and $\hat{S}_2$ are points of inflection. After these points, $G'(S)$ changes sign. Understanding these critical points is important to determine the directions of arrows for phase diagram analysis on plane (S, c).

The necessary condition for the existence of S_{high} is that $G\left(\widetilde{S}\right) > 0$ for $\exists \widetilde{S} < \bar{S}$. If this condition does not hold, we have a $G(S)$ function not crossing x-axis for the second

[25]Note that $\widetilde{S} > S_{low}$.

time and converging directly to z without changing sign. Then, there exists a unique equilibrium. Once the condition $G\left(\widetilde{S}\right) > 0$ is satisfied, we observe that the function $G\left(S\right)$ crosses unambiguously x-axis by S_{mid} and after tends to z when S approaches $\bar{S}$. With necessary conditions, we prove the existence of three steady states, one being unstable and two others being stable.

When there is no endogenous occurrence probability, the necessary condition (2.8) reduces to $R_{SS}\left(S\right) > 0$, which makes multiple equilibria an impossible outcome. Therefore, the model transforms into a standard neoclassical growth model. This completes the proof.

2.C Slope of the steady-state curve

By using the equation (A1.9) and implicit function theorem, we can find the slope of $\dot{c} = 0$ line.

$$\left.\frac{dc}{dS}\right|_{\dot{c}=0} = \frac{R_{SS}\left(S\right) - \frac{\bar{\psi} h_{SS}(S)}{u_c(R(S))}}{-\frac{u_{cc}(c) h_S(S) \bar{\psi}}{\left(u_c(c)\right)^2}} \tag{A1.10}$$

We can remark that the denominator is always negative. The sign of the necessary condition (2.8) is crucial in order to determine the sign of $\frac{dc}{dS}\big|_{\dot{c}=0}$. Since the nominator is quite similar to condition (2.8), it is easy to notice that nominator is unambiguously negative within zone A and C when $G_S\left(S\right) < 0$, which makes the slope of $\dot{c} = 0$ line positive at these areas. Within the zone B, since $G_S\left(S\right) > 0$, we cannot determine if the nominator is positive or negative. This is the reason why we draw two different phase diagrams.

2.D Proof of Lemma 1

The differential system describing the economy can be written as follows;

$$\begin{bmatrix} \dot{c} \\ \\ \dot{S} \end{bmatrix} = \begin{bmatrix} \frac{d\dot{c}}{dc} & \frac{d\dot{c}}{dS} \\ \\ \frac{d\dot{S}}{dc} & \frac{d\dot{S}}{dS} \end{bmatrix}_{\dot{c}=0,\dot{S}=0} \begin{bmatrix} c - c^* \\ \\ S - S^* \end{bmatrix}$$

$$\frac{d\dot{c}}{dc} = \rho - R_S(S) \tag{D.2}$$

$$\frac{d\dot{c}}{dS} = -\frac{u_c(c)}{u_{cc}(c)}\left[R_{SS}(S) - \frac{\bar{\psi}h_{SS}(S)}{u_c(c)}\right] \tag{D.3}$$

$$\frac{d\dot{S}}{dc} = -1 \tag{D.4}$$

$$\frac{d\dot{S}}{dS} = R_S(S) \tag{D.5}$$

We know that for a saddle-stable path system, it is necessary to have one positive and one negative eigenvalue, denoted $\mu_{1,2}$. As the $Tr(J) = \mu_1 + \mu_2$ and $Det(J) = \mu_1\mu_2$. It is sufficient to show that $Tr(J) > 0$ and $Det(J) < 0$. It is easy to see that $Tr(J) = \rho > 0$ and by arranging the terms for the determinant, we can see that determinant reduces to the multiple steady state condition $G(S)$. We conclude that $Det(J)$ is negative if

$$G_S(S) = R_{SS}(S) - \rho - \frac{\bar{\psi}h_{SS}(S)}{u_c(R(S))} + \frac{\bar{\psi}u_{cc}(R(S))}{(u_c(R(S)))^2} < 0 \tag{A1.11}$$

$$det(J) = (\rho - R_S(S))R_S(S) - \frac{u_c(c)}{u_{cc}(c)}\left[R_{SS}(S) - \frac{\bar{\psi}h_{SS}(S)}{u_c(c)}\right] \tag{A1.12}$$

Complex dynamics arise if $(Tr(J))^2 - 4Det(J) < 0$.

$$\rho^2 < 4\left[(\rho - R_S(S))R_S(S) - \frac{u_c(c)}{u_{cc}(c)}\left[R_{SS}(S) - \frac{\bar{\psi}h_{SS}(S)}{u_c(c)}\right]\right] \tag{A1.13}$$

As $Det(J)$ is shown to be negative for low and high steady states, this prevents these two steady states to have complex dynamics. However, for the middle steady state, there is a possibility to have complex dynamics if the condition above holds.

2.E Proof of Proposition 2

The third degree polynomial equation has the following form

$$a_1x^3 + b_1x^2 + c_1x + d_1 = 0 \tag{A1.14}$$

With the functional forms given in the text, we remark that terms $b_1 = -a_1$ with $a_1 = -2g\bar{\psi}\bar{h}$, $c_1 = -2g$ and $d_1 = g - \rho$. This simplifies the proof of the existence for three positive real roots. The discriminant of the cubic equation is the following :

$$\Delta = 18a_1b_1c_1d_1 - 4b_1^3d + b_1^2c_1^2 - 4_1ac_1^3 - 27a_1^2d_1^2 \tag{A1.15}$$

- $\Delta > 0$, the equation (A1.14) has three distinct real roots.
- $\Delta = 0$, the equation (A1.14) has a multiple root and all three roots are real.
- $\Delta < 0$, the equation (A1.14) has one real root and two non-real, complex roots.

Since we have $b_1 = -a_1$, we can reformulate the discriminant (A1.15) in the following way

$$\Delta = -b_1\left[4d_1b_1^2 - \left(c_1^2 - 18c_1d_1 - 27d_1^2\right)b_1 - 4c_1^3\right] \tag{A1.16}$$

Then, we have a second degree equation to be solved for the value of b. The discriminant of this second degree equation is $\Delta_1 = \left(c_1^2 - 18c_1d_1 - 27d_1^2\right)^2 - 64d_1c_1^3$. The equation (A1.16) is written as

$$\begin{aligned}\Delta = \underbrace{-b_1}_{<0}&\left[b_1 - \left(\frac{\left(c_1^2 - 18c_1d_1 - 27d_1^2\right) + \sqrt{\left(c_1^2 - 18c_1d_1 - 27d_1^2\right)^2 + 64d_1c_1^3}}{8d_1}\right)\right]\\ &\left[b_1 - \left(\frac{\left(c_1^2 - 18c_1d_1 - 27d_1^2\right) - \sqrt{\left(c_1^2 - 18c_1d_1 - 27d_1^2\right)^2 + 64d_1c_1^3}}{8d_1}\right)\right]\end{aligned} \tag{A1.17}$$

Then, by assuming $\Delta_1 > 0$, the discriminant Δ has a positive value if

$$\underbrace{\left(\frac{\left(c_1^2-18c_1d_1-27d_1^2\right)-\sqrt{\left(c_1^2-18c_1d_1-27d_1^2\right)^2+64d_1c_1^3}}{8d_1}\right)}_{L_E} < b_1 < \underbrace{\left(\frac{\left(c_1^2-18c_1d_1-27d_1^2\right)+\sqrt{\left(c_1^2-18c_1d_1-27d_1^2\right)^2+64d_1c_1^3}}{8d_1}\right)}_{U_E} \tag{A1.18}$$

where L_E and U_E are lower and upper extremes of the inequality (A1.18). By replacing the terms a_1, b_1, c_1, d_1 by their corresponding values, the condition (A1.18) becomes

$$\frac{\left(4g^2+36g\left(g-\rho\right)-27\left(g-\rho\right)^2\right)-\sqrt{\left(4g^2+36g\left(g-\rho\right)-27\left(g-\rho\right)^2\right)^2-2^9\left(g-\rho\right)}}{8\left(g-\rho\right)g\bar{h}} < \bar{\psi} < \frac{\left(4g^2+36g\left(g-\rho\right)-27\left(g-\rho\right)^2\right)+\sqrt{\left(4g^2+36g\left(g-\rho\right)-27\left(g-\rho\right)^2\right)^2-2^9\left(g-\rho\right)}}{8\left(g-\rho\right)g\bar{h}} \tag{A1.19}$$

However, the positive discriminant Δ proves only the existence of real roots and does not give an idea if these roots are positive or not. For this purpose, we use the Descartes rule. To see if the equation $G\left(S\right)=-2g\bar{\psi}\bar{h}S^3+2g\bar{\psi}\bar{h}S^2-2gS+\left(g-\rho\right)=0$, we write $G\left(-S\right)$

$$G\left(-S\right)=-2g\bar{\psi}\bar{h}\left(-S\right)^3+2g\bar{\psi}\bar{h}\left(-S\right)^2-2g\left(-S\right)+\left(g-\rho\right)=0 \tag{A1.20}$$

This yields

$$G\left(-S\right)=2g\bar{\psi}\bar{h}S^3+2g\bar{\psi}\bar{h}S^2+2gS+\left(g-\rho\right)=0 \tag{A1.21}$$

By assuming $g-\rho>0$, we observe that there is no sign change. This proves that all roots are positive for $G(S)$. This completes the proof.

2.F The economy with adaptation and mitigation

Adaptation cost a social disutility

We write down the Hamilton-Jacobi-Bellman equation for the economy implementing both adaptation and mitigation policies.

$$\begin{aligned}\rho V^B(S, K_A) = \max_c \big\{ & u(c) - Q^1(A) - Q^2(M) + V_S^B(S, K_A)(R(S) + \Gamma(M) - c) \\ & + V_{K_A}^B(S, K_A)(A - \delta K_A) - h(S)\left(V^B(S) - \varphi(S, K_A)\right)\big\}\end{aligned} \tag{A1.22}$$

where $V^B(S)$ is the value of the maximisation program before the event. As also stated in the text, the value of the problem after the event is as follows:

$$\varphi(S, K_A) = V^B(S, K_A) - \psi(K_A) \tag{A1.23}$$

The economy is exposed to an inflicted damage after the event. The first-order conditions are given by

$$u_c(c) = V_S^B(S, K_A) \tag{A1.24}$$

$$Q_A^1(A) = V_{K_A}^B(S, K_A) \tag{A1.25}$$

$$Q_M^2(M) = V_S^B(S, K_A) \tag{A1.26}$$

Computing the derivative of A1.22 with respect to the environmental quality stock $S(t)$

and adaptation capital K_A yields

$$\rho V_S^B(S,K_A) = -h_S(S)\,\psi(K_A) + V_{SS}^B(S,K_A)\big(R(S) + \Gamma(M) - Q^1(A) - c\big) + V_{K_AS}^B(S,K_A)(A - \delta K_A) + V_S^B(S,K_A)\,R_S(S) \quad \text{(A1.27)}$$

$$\rho V_{K_A}^B(S,K_A) = V_{SK_A}^B(S,K_A)\big(R(S) + \Gamma(M) - Q^1(A) - c\big) + V_{K_AK_A}^B(S,K_A)(A - \delta K_A) - \delta V_{K_A}^B(S,K_A) - h(S)\,\psi_{K_A}(K_A) \quad \text{(A1.28)}$$

Differentiating the equation (A1.24) and using equations (A1.24) and (A1.27) gives

$$\frac{u_{cc}(c)}{u_c(c)}\dot{c} = \frac{V_{SS}^B(S,K_A)}{V_S^B(S,K_A)}\dot{S} \quad \text{(A1.29)}$$

$$\frac{Q_{AA}^1(A)}{Q_A^1(A)}\dot{A} = \frac{V_{K_AK_A}^B(S,K_A)}{V_{K_A}^B(S,K_A)}\dot{K_A}$$

Using the equations (A1.27) and (A1.28), at the steady-state, we have

$$R_S(S) - \rho - \frac{\psi(K_A)\,h_S(S)}{V_S^B(S)} = 0 \quad \text{(A1.30)}$$

$$\rho + \delta + \frac{h(S)\,\psi_{K_A}(K_A)}{Q_A^1(\delta K_A)} = 0 \quad \text{(A1.31)}$$

Using the first order conditions (A1.24), (A1.25), (A1.26) and $R(S) + \Gamma(M) - Q^1(A) = c$, we write inverse functions as

$$R^{-1}(c - \Gamma(M)) = S \quad \text{(A1.32)}$$

$$c = u_c^{-1}\left(\frac{P_M}{\Gamma_M\left(M\right)}\right) \tag{A1.33}$$

Plugging the equations (A1.32) and (A1.33) in (A1.31), we have

$$\rho + \delta + \frac{h\left(R^{-1}\left(u_c^{-1}\left(\frac{P_M}{\Gamma_M(M)}\right) - \Gamma\left(M\right)\right)\right)\psi_{K_A}\left(K_A\right)}{Q_A^1\left(\delta K_A\right)} = 0 \tag{A1.34}$$

Using the implicit function theorem for (A1.34) yields the equation (2.21).

Adaptation cost in the resource constraint

In this section, we model the adaptation cost in the resource constraint. The Hamilton-Jacobi-Bellman equation is the following

$$\begin{aligned}\rho V^B\left(S, K_A\right) = \max_c \big\{&u\left(c\right) - Q^2\left(M\right) + V_S^B\left(S, K_A\right)\left(R\left(S\right) + \Gamma\left(M\right) - Q^1\left(A\right) - c\right)\\ &+V_{K_A}^B\left(S, K_A\right)\left(A - \delta K_A\right) - h\left(S\right)\left(V^B\left(S\right) - \varphi\left(S, K_A\right)\right)\big\}\end{aligned} \tag{A1.35}$$

The first-order conditions are given by

$$u_c\left(c\right) = V_S^B\left(S, K_A\right) \tag{A1.36}$$

$$Q_A^1\left(A\right)V_S^B\left(S, K_A\right) = V_{K_A}^B\left(S, K_A\right) \tag{A1.37}$$

$$Q_M^2\left(M\right) = V_S^B\left(S, K_A\right) \tag{A1.38}$$

Computing the derivative of A1.35 with respect to the environmental quality stock $S\left(t\right)$ and adaptation capital K_A yields

$$\rho V_S^B(S,K_A) = -h_S(S)\,\psi(K_A) + V_{SS}^B(S,K_A)\left(R(S)+\Gamma(M)-c-Q^1(A)\right) + V_{K_AS}^B(S,K_A)(A-\delta K_A) + V_S^B(S,K_A)\,R_S(S) \quad \text{(A1.39)}$$

$$\rho V_{K_A}^B(S,K_A) = V_{SK_A}^B(S,K_A)\left(R(S)+\Gamma(M)-c-Q_A^1(A)\right) + V_{K_AK_A}^B(S,K_A)(A-\delta K_A) - \delta V_{K_A}^B(S,K_A) - h(S)\,\psi_{K_A}(K_A) \quad \text{(A1.40)}$$

Differentiating the equation (A1.36) and using equations (A1.36) and (A1.39) gives

$$\frac{u_{cc}(c)}{u_c(c)}\dot{c} = \frac{V_{SS}^B(S,K_A)}{V_S^B(S,K_A)}\dot{S} \quad \text{(A1.41)}$$

$$\frac{Q_{AA}^1(A)}{Q_A^1(A)}\dot{A} = \frac{V_{K_AK_A}^B(S,K_A)}{V_{K_A}^B(S,K_A)}\dot{K}_A$$

Using the equations (A1.39) and (A1.40), at the steady-state, we have

$$R_S(S) - \rho - \frac{\psi(K_A)\,h_S(S)}{V_S^B(S)} = 0 \quad \text{(A1.42)}$$

$$\rho + \delta + \frac{h(S)\,\psi_{K_A}(K_A)}{Q_A^1(A)\,u_c(c)} = 0 \quad \text{(A1.43)}$$

Using the first order conditions (A1.36), (A1.37), (A1.38) and $R(S)+\Gamma(M)=c$, we write inverse functions as

$$R^{-1}\left(c+Q^1(A)-\Gamma(M)\right) = S \quad \text{(A1.44)}$$

$$c = u_c^{-1}\left(\frac{P_M}{\Gamma_M\left(M\right)}\right) \tag{A1.45}$$

Plugging the equations (A1.44) and (A1.45) in (A1.43), we have

$$\rho + \delta + \frac{h\left(R^{-1}\left(u_c^{-1}\left(\frac{P_M}{\Gamma_M(M)}\right) - \Gamma\left(M\right) + Q^1\left(A\right)\right)\right)\psi_{K_A}\left(K_A\right)}{Q_A\left(\delta K_A\right)u_c\left(c\right)} = 0 \tag{A1.46}$$

In order to see whether adaptation and mitigation are complementary or substitutes, we compute the derivatives of the implicit function (A1.46) which yield the equation (2.22).

2.G Proof of Proposition 3 (Adaptation)

The equation (2.7) combined with (2.1) at the steady state can be reformulated as

$$G\left(S\right) = -\bar{\psi}\bar{h}gS^2 + \left(\bar{\psi}\bar{h}g - 2g\right)S + \left(g - \rho\right) = 0 \tag{A1.47}$$

It is easy to show that this equation has one positive and one negative real root:

$$S_{1,2} = \frac{\left(\bar{\psi}\bar{h}g - 2g\right) \pm \sqrt{\left(\bar{\psi}\bar{h}g - 2g\right)^2 + 4\bar{\psi}\bar{h}g\left(g - \rho\right)}}{2\bar{\psi}\bar{h}g} \tag{A1.48}$$

We exclude the negative root since it has no economic meaning. Consequently, the economy has a unique equilibrium without any possibility of multiple equilibria. What happens if this economy starts to invest in adaptation capital? For the sake of analytical tractability, we use a linear hazard and adaptation function. In the numerical analysis, we relax the linearity assumption on functional forms and use more general functional forms to show the robustness of our results.

$$\psi\left(K_A\right) = \bar{\psi}f\left(K_A\right) = \bar{\psi}\left(1 - aK_A\right) \tag{A1.49}$$

$$Q\left(A\right) = \phi\frac{A^2}{2} \tag{A1.50}$$

where a shows at which extent the adaptation capital is able to decrease the vulnerability against the inflicted damage $\bar{\psi}$. For the sake of analytical tractability, we take $f(K_A)$ as a linear function. In the numerical analysis, we relax the linearity assumption. ϕ stands for a scale parameter for the adaptation cost function. The benchmark economy with an adaptation policy ends up with the following $G(S)$ (see Appendix 2.J)

$$G(S) = -\bar{h}gzS^3 + \left(\left(g+\bar{h}g\right)z - \bar{\psi}\bar{h}g\right)S^2 + \left(\bar{\psi}\bar{h}g - 2g - zg\right)S + (g-\rho) = 0 \tag{A1.51}$$

where $z = \frac{\bar{h}\left(a\bar{\psi}\right)^2}{\phi\delta(\rho+\delta)}$. For analytical tractability, we suppose that $\bar{\psi} = \frac{z}{\bar{h}}$. Then, the equation (A1.51) can be reformulated in a simplified form

$$G(S) = -\bar{h}gzS^3 + \bar{h}gzS^2 - 2gS + (g-\rho) = 0$$

This equation is similar to (2.13) which can be shown to have three positive real roots. Following the same method for the proof of Proposition 2., the condition to have multiple equilibria in the benchmark economy augmented by the adaptation investments is as follows[26]

$$\begin{aligned}&\frac{\left[\left(4g^2+36g(g-\rho)-27(g-\rho)^2\right) - \sqrt{\left(4g^2+36g(g-\rho)-27(g-\rho)^2\right)^2 - 2^9g^3(g-\rho)}\right]}{8(g-\rho)g\left(\bar{h}\right)^2}\\ &< \bar{\psi} < \frac{\left[\left(4g^2+36g(g-\rho)-27(g-\rho)^2\right) + \sqrt{\left(4g^2+36g(g-\rho)-27(g-\rho)^2\right)^2 - 2^9g^3(g-\rho)}\right]}{8(g-\rho)g\left(\bar{h}\right)^2}\end{aligned} \tag{A1.52}$$

[26]The proof is available upon request.

2.H The economy with only mitigation

We write down the Hamilton-Jacobi-Bellman equation for the economy with a mitigation activity.

$$\rho V^B(S) = \max_c \left\{ u(c) - Q^2(M) + V_S^B(S)(R(S) + \Gamma(M) - c) - h(S)\left(V^B(S) - \varphi(S)\right) \right\} \tag{A.1}$$

where $V^B(S)$ is the value of the maximisation program before the event. As also stated in the text, the value of the problem after the event is the following:

$$\varphi(S) = V^B(S) - \bar{\psi} \tag{A1.53}$$

The economy is exposed to an inflicted damage after the event. The first-order conditions are given by

$$u'(c) = V_S^B(S) \tag{A1.54}$$

$$Q_M^2 = V_S^B(S)\,\Gamma_M(M) \tag{A1.55}$$

Computing the derivative of (A.1) with respect to the environmental quality stock $S(t)$ yields

$$\rho V_S^B(S) = -\bar{\psi} h_S(S) + V_{SS}^B(S)(R(S) + \Gamma(M) - c) + V_S^B(S)\,R_S(S) \tag{A1.56}$$

Differentiating the equation (A1.54) and using equations (A1.54) and (A1.56) gives

$$\frac{u_{cc}(c)}{u_c(c)}\dot{c} = \frac{V_{SS}^B(S)}{V_S^B(S)}\dot{S} \tag{A1.57}$$

$$\rho = -\frac{\bar{\psi} h_S(S)}{V_S^B(S)} + \frac{V_{SS}^B(S)}{V_S^B(S)}\dot{S} + R_S(S) \tag{A1.58}$$

Arranging equations (A1.57) and (A1.58) gives the Keynes-Ramsey rule

$$\dot{c}^{RE} = -\frac{u_c(c)}{u_{cc}(c)}\left[R_S(S) - \rho - \frac{\bar{\psi} h_S(S)}{u_c(c)}\right]$$

At the steady-state, we have

$$G(S) = R_S(S) - \rho - \frac{\bar{\psi} h_S(S)}{u_c(R(S) + \Gamma(M))} \tag{A1.59}$$

Note that the optimal steady state level of mitigation M depends on S. Differentiating the equation (A1.59) with respect to S yields

$$G_S(S) = R_{SS}(S) - \frac{\bar{\psi} h_{SS}(S)}{u_c(c)} + \frac{\bar{\psi} h_S(S)\, u_{cc}}{(u_c(c))^2}\left(R_S(S) + \Gamma_M(M)\frac{dM}{dS}\right)$$

2.I Proof of Proposition 4 (Mitigation)

We start the proof by solving the following optimisation problem with mitigation policy. Similar to the resolution of the benchmark economy, we write the Hamilton-Jacobi-Bellman equation

$$\rho V^B(S) = \max_c \left\{u(c) + V_S^B(S)\left(R(S) + \mathit{\Gamma}(M) - Q_2(M) - c\right) - h(S)\left(V^B(S) - \varphi(S)\right)\right\} \tag{A.1}$$

where $V^B(S)$ is the value of the maximisation program before the event. The economy is exposed to a constant inflicted damage after the occurrence of the event. As also stated in the text, the value of the problem after the event is the following

$$\varphi(S) = V^B(S) - \bar{\psi} \tag{A1.60}$$

The first-order conditions are given by

$$u'(c) = V_S^B(S) \tag{A.2}$$

$$\mathit{\Gamma}_M(M) = Q_M^2(M) = P_M \tag{A1.61}$$

Using the envelop theorem, we have

$$\rho V_S^B(S) = -\bar{\psi} h_S(S) + V_{SS}^B(S)\left(R(S) + \mathit{\Gamma}(M) - Q_2(M) - c\right) + V_S^B(S) R_S(S) \tag{A1.62}$$

Differentiating the equation (A.2) and using equations (A.2) and (A1.62) gives

$$\frac{u_{cc}(c)}{u_c(c)}\dot{c} = \frac{V_{SS}^B(S)}{V_S^B(S)}\dot{S} \tag{A1.63}$$

$$\rho = -\frac{\bar{\psi} h_S(S)}{V_S^B(S)} + \frac{V_{SS}^B(S)}{V_S^B(S)}\dot{S} + R_S(S) \tag{A1.64}$$

Arranging equations (A1.63) and (A1.64) gives the Keynes-Ramsey rule

$$\dot{c}^{RE} = -\frac{u_c(c)}{u_{cc}(c)}\left[R_S(S) - \rho - \frac{\bar{\psi} h_S(S)}{u_c(c)}\right]$$

At the steady state, we have

$$G(S) = R_S(S) - \rho - \frac{\bar{\psi} h_S(S)}{u_c\left(R(S) + \mathit{\Gamma}(M) - Q_2(M)\right)} = 0$$

Since we use linear specifications for analytical tractability, we have the optimum level of mitigation investment as $M^* = \frac{1}{P_M}$. Then, we can reformulate the function $G(S)$

$$G(S) = -2g\bar{\psi}\bar{h}S^3 + 2g\bar{\psi}\bar{h}S^2 + \left(\frac{\bar{\psi}\bar{h}}{P_M} - 2gS\right) + (g - \rho) = 0 \tag{A1.65}$$

where $a_1 = -2g\bar{\psi}\bar{h}$, $b_1 = 2g\bar{\psi}\bar{h}$, $c_1 = \frac{\bar{\psi}\bar{h}}{P_M} - 2gS$ and $d_1 = g - \rho$. The equation (A1.65) is similar to $G(S)$ of the benchmark economy without an environmental policy. The only term that is different in the economy with mitigation policy with respect to the benchmark economy is the term c_1 of the equation (A1.14). The term c_1 is higher in the case with mitigation policy with respect to the benchmark economy case.

We look at the effect of a higher value c_1 on the upper and lower extremes of the inequality (A1.18). By assuming $c_1 < 0$ and $d_1 > 0$, the derivative of the upper and lower extremes are as follows

$$\frac{\partial L_E}{\partial c_1} = \frac{1}{2}\frac{\underbrace{\left(c_1^2 - 18c_1d_1 - 27d_1^2\right)}_{>0}\underbrace{\left(2c_1 - 18d_1\right)}_{<0}}{\left(\left(c_1^2 - 18c_1d_1 - 27d_1^2\right)^2\right)^{\frac{1}{2}}} - \frac{1}{2}\left(\frac{\underbrace{\left(c_1^2 - 18c_1d_1 - 27d_1^2\right)}_{>0}\underbrace{\left(2c_1 - 18d_1\right)}_{<0} + 3.64d_1c_1^2}{\left(\left(c_1^2 - 18c_1d_1 - 27d_1^2\right)^2 + 64d_1c_1^3\right)^{\frac{1}{2}}}\right) > 0$$

$$\frac{\partial U_E}{\partial c_1} = \frac{1}{2}\frac{\underbrace{\left(c_1^2 - 18c_1d_1 - 27d_1^2\right)}_{>0}\underbrace{\left(2c_1 - 18d_1\right)}_{<0}}{\left(\left(c_1^2 - 18c_1d_1 - 27d_1^2\right)^2\right)^{\frac{1}{2}}} + \frac{1}{2}\left(\frac{\underbrace{\left(c_1^2 - 18c_1d_1 - 27d_1^2\right)}_{>0}\underbrace{\left(2c_1 - 18d_1\right)}_{<0} + 3.64d_1c_1^2}{\left(\left(c_1^2 - 18c_1d_1 - 27d_1^2\right)^2 + 64d_1c_1^3\right)^{\frac{1}{2}}}\right) < 0$$

We observe that the occurrence of multiple equilibria with mitigation policy is less likely, since there is a smaller range of values for damage $\bar{\psi}$ that causes multiple equilibria.

2.J Derivation of the equation (A1.51)

The Hamilton-Jacobi-Bellman equation of the economy with only adaptation policy is the following

$$\begin{aligned}\rho V^B\left(S, K_A\right) = \max_c \big\{ & u\left(c\right) - Q^1\left(A\right) + V_S^B\left(S, K_A\right)\left(R\left(S\right) - c\right) \\ & + V_{K_A}^B\left(S, K_A\right)\left(A - \delta K_A\right) - h\left(S\right)\left(V^B\left(S, K_A\right) - \varphi\left(S, K_A\right)\right)\big\} \quad \text{(A1.66)}\end{aligned}$$

First order conditions for an internal optimal solution give

$$u_c = V_S^B \tag{A.2}$$

$$V_{K_A}^B = Q_A^1 V_S^B \tag{E.3}$$

$$\rho V_S^B = V_{SS}^B \left(R\left(S\right) - c\right) + V_S^B R_S + V_{K_A S}\left(A - \delta K_A\right) - h_S\left(S\right)\psi\left(K_A\right) \tag{A1.67}$$

where $V_{K_A S} = \frac{\partial^2 V}{\partial K_A \partial S}$

$$\rho V_{K_A}^B = V_{SK_A}^B \left(R\left(S\right) - c\right) + V_S^B R_S + V_{K_A K_A}\left(A - \delta K_A\right) - h\left(S\right)\psi_{K_A}\left(K_A\right) - \delta V_{K_A}^B \tag{A1.68}$$

The optimal dynamics of consumption and adaptation investment are

$$\dot{c}^{RE} = -\frac{u_c\left(c\right)}{u_{cc}\left(c\right)}\left[R_S\left(S\right) - \rho - \frac{\psi\left(K_A\right)h_S\left(S\right)}{u_c\left(c\right)}\right] \tag{A1.69}$$

$$\dot{A} = \frac{Q_A^1\left(A\right)}{Q_{AA}^1\left(A\right)}\left[\left(\rho + \delta\right) + \frac{h\left(S\right)\psi_{K_A}\left(K_A\right)}{Q_A}\right] \tag{A1.70}$$

Using $A = \delta K_A$, at the steady state, we have

$$Q_A^1\left(\rho + \delta\right) + h\left(S\right)\psi_{K_A}\left(K_A\right) = 0 \tag{A1.71}$$

$$R_S\left(S\right) - \rho - \frac{\psi\left(K_A\right)h_S\left(S\right)}{u_c\left(c\right)} = 0 \tag{A1.72}$$

Using the equations (A1.69) and $R\left(S\right) = c$, we have

$$G\left(S\right)=R_{S}\left(S\right)-\rho-\frac{\psi\left(K_{A}\right)h_{S}\left(S\right)}{u_{c}\left(R\left(S\right)\right)} \tag{A1.73}$$

Differentiating the equation (A1.73) with respect to S yields

$$G_{S}\left(S\right)=R_{SS}\left(S\right)-\frac{h_{SS}\left(S\right)\psi\left(K_{A}\right)}{u_{c}\left(R\left(S\right)\right)}+\frac{h_{S}\left(S\right)\psi\left(K_{A}\right)u_{cc}\left(R\left(S\right)\right)R_{S}\left(S\right)}{\left(u_{c}\left(R\left(S\right)\right)\right)^{2}}\underbrace{-\left[\frac{\psi_{K_{A}}\left(K_{A}\right)h_{S}\left(S\right)}{u_{c}\left(R\left(S\right)\right)}\right]\frac{dK_{A}}{dS}}_{=Z_{1}>0}>0$$

It is also possible to write the equation (A1.73) by using the functional forms in (2.2.2). Using the equations (A1.69), (A1.70) and functional forms presented in the section (2.2.2), the function $G\left(S\right)$ that describes the steady-state of the economy becomes

$$G\left(S\right)=\underbrace{g\left(1-2S\right)-\rho+\frac{\bar{\psi}\bar{h}}{u_{c}}}_{Benchmark}+\underbrace{\frac{a\bar{\psi}\left(1-\bar{h}S\right)}{\phi\delta\left(\rho+\delta\right)u_{c}}}_{Adaptation} \tag{A1.74}$$

The first part for the expression (A1.74) is the same as in the benchmark model. The additional last term comes with the adaptation policy. After arranging terms, the equation (A1.74) writes

$$G\left(S\right)=-\bar{h}gzS^{3}+\left(\left(g+\bar{h}g\right)z-\bar{\psi}\bar{h}g\right)S^{2}+\left(\bar{\psi}\bar{h}g-2g-zg\right)S+\left(g-\rho\right)=0 \tag{A1.75}$$

Chapter 3

Creative Destruction vs Destructive Destruction : A Schumpeterian Approach for Adaptation and Mitigation

Can Askan Mavi

3.1 Introduction

In this chapter, we take a step further to answer the following questions : How does the catastrophic event probability affect the creative destruction process in the economy? What is the effect of pollution tax on the growth rate and the implications of catastrophe probability regarding this effect? How does the market adjust the equilibrium level of adaptation and mitigation when it faces a higher catastrophe probability?

Many recent reports (see EU-Innovation (2015) Road map for Climate Services [1]) high-

[1]The definition for climate services given in this report is the following : "We attribute to the term a broad meaning, which covers the transformation of climate-related data — together with other relevant in-

light the importance to build a market economy through R&D innovations that handles adaptation and mitigation services to create a low carbon and climate-resilient economy. Climate services market aims at providing the climate knowledge to the society through informational tools.[2] These services involve a very detailed analysis of the existing environmental knowledge and R&D activity that inform the society about the impacts of the climate change. In addition, these services give necessary information to take action against extreme events. To summarize, one can say that the purpose of climate services is to bridge the innovation with entrepreneurship that could create new business opportunities and market growth.

"Significantly strengthening the market for climate services towards supporting the building of Europe's resilience to climate change and its capacity to design a low-carbon future will require targeted research and innovation investments. These investments are required to provide the evidence, knowledge and innovations that would identify opportunities, and explore and deliver the means for fuelling the growth of this market." (EU-Innovation (2015) Road map for Climate Services, p.19)

Indeed, it is interesting to see the words "service" and "market" for adaptation and mitigation activities since the existing literature has studied adaptation and mitigation policies in a social optimum and not in a market economy framework (Zemel (2015), Bréchet et al. (2012), Tsur and Zemel (2016a)).

In recent years, climate change started to be considered as a business opportunity[3] since companies could develop a new service or product to adapt to catastrophic events. These products and services are expected to ensure competitiveness and advantage for companies on the market which promote the growth. Regarding this recent evolution about adaptation and mitigation activities, a decentralized market analysis is more than necessary to analyze rigorously the long term implications of adaptation and mitigation.

formation — into customized products such as projections, forecasts, information, trends, economic analysis, assessments (including technology assessment), counselling on best practices, development and evaluation of solutions and any other service in relation to climate that may be of use for the society at large. As such, these services include data, information and knowledge that support adaptation, mitigation and disaster." (EU-Innovation (2015) - A European Research and Innovation Roadmap for Climate Services, Box1. pp 10.)

[2]An example can be a smartphone application that informs farmers about weather and how to proceed in extreme weather events.

[3]see European Commission- Road map for Climate Services 2015 and National and Sub-national Policies and Institutions. In: Climate Change 2014: Mitigation of Climate Change.

The world faces undesirable extreme events entailing significant environmental damages. Our aim in this chapter is to see how adaptation and reducing the pollution sources (mitigation)[4] can be possible through the R&D activity handled by the market economy exposed to an abrupt event. To our knowledge, there are no research studying the adaptation and mitigation activities in a decentralized framework with taking into account the uncertainty about catastrophic events. Our contribution relies on building a decentralized growth model that analyzes adaptation and mitigation policies. Moreover, existing studies examine these policies on exogenous growth models and endogenous technological progress is a missing component (see Zemel (2015), Bréchet et al. (2012), Tsur and Zemel (2016a), Tsur and Withagen (2012), de Zeeuw and Zemel (2012)). In a sense, our study is the first anayltical framework that focuses on adaptation and mitigation through an endogenous R&D process.

Firstly, the model in this chapter builds on the literature on adaptation and mitigation (Bréchet et al. (2012)) and also includes the uncertainty about abrupt climate event the effects (see Tsur and Zemel (1996), Tsur and Zemel (1998)) Secondly, our model belongs to Schumpeterian growth literature which started with the seminal chapter of Aghion and Howitt (1992).

To inform the reader about adaptation and mitigation analysis, Bréchet et al. (2012), Kama and Pommeret (2016), Kane and Shogren (2000) and Buob and Stephan (2011) are first analytical studies that treat the optimal design of adaptation and mitigation. However, these studies focusing on trade-offs between adaptation and mitigation neglect the uncertainty about abrupt climate events. To fill this gap in the literature, Zemel (2015) and Tsur and Zemel (2016a) introduce Poisson uncertainty in Bréchet et al. (2012) framework and show that a higher catastrophic event probability induces more adaptation capital at long-run.

Now, we return to the Schumpeterian growth literature. Very first study that combines the environment and Schumpeterian growth models is Aghion and Howitt (1997). Authors introduce the pollution in a Schumpeterian growth and make a balanced growth path analysis by taking into account the sustainable development criterion[5]. Grimaud (1999) extends this model to a decentralized economy in which he implements the optimum by R&D subsidies

[4]In this study, R&D aims at decreasing the pollution intensity of machines used for final good production.

[5]The sustainable development criterion requires that the utility from consumption follows a constant or an increasing path at the long run. i.e, $\frac{du(c)}{dt} \geq 0$

and pollution permits.

One of the first attempts to model environmental aspects in a Schumpeterian growth model is Hart (2004). He studies the effect of a pollution tax and finds that environmental policy can be a win-win policy by increasing the pollution intensity and promoting the growth rate at the long run. In the same line, Ricci (2007) shows in a Schumpeterian growth model that long-run growth of the economy is driven by the knowledge accumulation. In his model, environmental regulation pushes the final good producers to use cleaner vintages. The important difference between Hart (2004) and Ricci (2007) is that the latter treats a continuum of different vintages. However, Hart (2004) proposes a model in which there are only two young vintages on sale. Due to this modeling difference, Ricci (2007) shows that tightening environmental policy does not foster the economic growth since the marginal contribution of R&D to economic growth falls. However, uncertainty about abrupt climate events is totally overlooked in these models. One of the focus of this study is to analyze how the results stated by Hart (2004) and Ricci (2007) alter with respect to a catastrophic event possibility.

In this chapter, differently from Hart (2004) and Ricci (2007), the benefit of R&D is twofold ; firstly, with the assumption that wealthier countries resist more easily to catastrophic events (see Mendhelson et al. (2006)), we show that investing in R&D increases the wealth of the economy and make it more resilient to catastrophic events. The knowledge serves as a tool of adaptation only if the abrupt event occurs. In this sense, knowledge plays also a proactive role for adaptation[6]. Secondly, R&D decreases the pollution intensity of intermediate goods (i.e, mitigation) as in Ricci (2007) and increases the total productivity which allows a higher growth rate at the balanced growth path.

In this chapter, we show that there are two opposite effects of catastrophe probability on the creative destruction rate. A first channel is straightforward, a higher abrupt event probability increases the impatience level of agents. It follows that the interest rate in the market tends to increase. Consequently, the expected value of an R&D patent decreases as well, since the labor allocation in this sector. This one can be called *discount effect*.

The second channel is more interesting : when the abrupt event probability is higher, the marginal benefit of R&D activity increases since the knowledge stock helps to increase the

[6]See Zemel (2015) for a detailed discussion about proactive adaptation policy.

resilience of the economy against the inflicted penalty due to an abrupt event. Consequently, the interest rate in the market decreases and the expected value of R&D patents increases. This one can be called *adaptation effect*.

In other words, the more the hazard rate increases, the more the opportunity cost of not investing in R&D increases. In a nutshell, a higher hazard rate pushes the economy to invest more in R&D activity. We show that when the catastrophic damage exceeds some threshold, an increase in catastrophe probability boosts the creative destruction rate in the economy. This is due to the fact that adaptation effect dominates the discount effect.

Our results indicate that market's adaptation level relative to mitigation level depends on the ratio between the pollution intensity of intermediate goods and total productivity rate of labor. In addition, the more the R&D sector offers cleaner intermediate goods, the less the economy adapts to abrupt climate damages. This relies on the usual assumption that cleaner intermediate goods are less productive. Then, with cleaner intermediate goods, there is a lower growth rate and knowledge accumulation. Indeed, the trade-off between adaptation and mitigation (see Bréchet et al. (2012)) is not present in our model since the growth rate of the productivity and the decrease of the emission intensity come from the same source which is the R&D sector. In other words, the economy increases both adaptation and mitigation at each date. Interestingly, there is a new trade-off between adaptation and pollution that can arise in the economy. R&D activity decreases the pollution intensity but at the same time, it seeks to increase the total productivity of the economy. Then, the scale of the economy increases with R&D activity. If the scale effect dominates the emission intensity decrease, the growth rate increases. However, in this case, the pollution growth is higher even with cleaner intermediate goods since the scale of the economy increases. This is close to the so-called Jevons Paradox which states that technological improvements increase energy efficiency but lead to a higher pollution in the long term[7].

Before coming to pollution tax analysis, it is worthwhile to note that firms mitigate, since they face a pollution tax levied on the use of polluting intermediate goods. Hence, they are investing in R&D to decrease pollution intensity in order to lower the tax burden. Our model shows a positive effect of a pollution tax effect on growth as in Ricci (2007) since the lower demand for intermediate goods implies a shift of labor from final good sector to

[7] A similar result is shown empirically for the case of India by Ollivier and Barrows (2016).

R&D sector which promotes the economic growth. Differently from Ricci (2007), we show that a higher hazard rate can increase the positive effect of green tax burden on growth rate of the economy at long run, if penalty rate is sufficiently high. This effect is due to a higher marginal benefit of R&D since it helps an economy to show a better resistance to catastrophic events.

The remainder of the chapter is organized as follows. Section 2 presents the decentralized model while section 3 focuses on the balanced growth path analysis. In section 4, we examine the adaptation and mitigation handled by the market economy and next, in section 5 we study the welfare implications of green tax burden and abrupt event probability. Section 6 concludes.

3.2 Model

We make an extension of the Schumpeterian model of endogenous growth (Aghion and Howitt (1997)) to consider the effect of uncertain abrupt climate events on the market economy. Our model also adds an environmental aspect to Aghion and Howitt (1997) since the production emits pollutants (see Hart (2004), Ricci (2007)). The production is realized in three stages. First, labor is used in R&D sector to improve the productivity of intermediate goods. Pollution intensity is also a technological variable since successful innovations decrease the emission intensity of intermediate goods. Second, the machines (intermediate goods) are supplied by a monopolistic intermediate good producer because the technology that allows the production of machines is protected by patents. Their production emits pollutants which is imposed by a tax set by the policymaker. Third, the final good is produced by combining intermediate good and labor allocated by the household who faces an abrupt event probability. The possibility of an abrupt event changes the labor allocation decisions of the household.

3.2.1 Production of Final Good

An homogeneous final good is produced using labor, L_Y , intermediate good x, according to aggregate production function (see Stokey (1998) and Ricci (2007)).

$$Y(t) = L_Y(t)^{1-\alpha} \int_0^1 \phi(v,t)\, z(v,t)\, x(v,t)^{\alpha}\, dv \tag{3.1}$$

where t is the continuous time index. The parameter α stands for the elasticity of intermediate good in the production function. There exists a continuum of different technologies available on the market indexed by $v \in [0,1]$. $\phi(v,t)$ is the technology level that can be referred to as the implicit labor productivity index. The important novelty in the production function with respect to Aghion and Howitt (1997) framework is that the emission intensity $z(v,t)$ of intermediate good is heterogeneous across firms which is defined by

$$z(v,t) = \left(\frac{P(v,t)}{\phi(v,t)^{\frac{1}{\beta}}\, x(v,t)} \right)^{\alpha\beta} \tag{3.2}$$

where $P(v,t)$ represents the polluting emissions of a given firm. The term $\alpha\beta$ is the share of pollution in the production function (see Appendix 3.A). The emission intensity variable z is defined in a close manner to Stokey (1998) since pollution enters as an input in production function and reducing its use decreases the production.

From equation (3.2), the aggregate pollution stemming from the production of intermediate goods can be written as

$$P(t) = \int_0^1 P(v,t) = \int_0^1 (z(v,t))^{\frac{1}{\alpha\beta}}\, \phi(v,t)^{\frac{1}{\beta}}\, x(v,t)\, dv$$

Contrary to Stokey (1998) and Aghion and Howitt (1997), R&D activity changes progressively the pollution intensity at the long term which is heterogeneous across firms in the economy (Ricci (2007)). Unlike Stokey (1998) and Aghion and Howitt (1997), we can remark that the productivity of intermediate goods does not depend only on the labor productivity index ϕ but also on the pollution intensity z.

3.2.2 Final Good Producer's Program

By using the production function (3.1), the instantaneous profit of competitive firms is

$$\underset{x(v,t),L_Y(t)}{max} \psi\left(t\right) = Y\left(t\right) - \int_0^1 p\left(v,t\right) x\left(v,t\right) dv - w\left(t\right) L_Y\left(t\right) \tag{3.3}$$

where $p\left(v,t\right)$ and $w\left(t\right)$ are the price of intermediate good and wage respectively. The final good sector is in perfect competition and the price of the final good is normalized to one. From the maximization program, we write the demand of intermediate good and labor of final good producer

$$p\left(v,t\right) = \alpha\phi\left(v,t\right) z\left(v,t\right) \left(\frac{L_Y\left(t\right)}{x\left(v,t\right)}\right)^{1-\alpha} \tag{3.4}$$

$$w\left(t\right) = \left(1-\alpha\right) \int_0^1 \left(\phi\left(v,t\right)\right) \left(\frac{x\left(v,t\right)}{L_Y\left(t\right)}\right)^{\alpha} dv = \left(1-\alpha\right) \frac{Y\left(t\right)}{L_Y\left(t\right)} \tag{3.5}$$

When the final good producer maximizes its instantaneous profit, it takes the technology level as given.

3.2.3 Intermediate Good Producer's Program

The intermediate good producer is a monopolist. It faces a factor demand (3.4) and offers intermediate good to the final good sector. The cost of providing intermediate goods implies foregone production which is subtracted from the consumption (see Nakada (2010)). The intermediate good producer faces a green tax $h\left(t\right)$ levied on the use of polluting machines. The maximization program of intermediate good producer is ;

$$\underset{x(v,t)}{max} \pi\left(t\right) = p\left(v,t\right) x\left(v,t\right) - \chi x\left(v,t\right) - h\left(t\right) P\left(v,t\right) \tag{3.6}$$

where χ stands for the constant marginal cost of producing intermediate goods (Acemoglu and Cao (2015)). In the absence of the green tax, market economy will not have incentive to decrease pollution intensity (i.e mitigation) by the means of R&D activity. As pollution enters in the maximization program of the intermediate good producer as a cost, there are incentives to make R&D[8] to reduce this cost.

We write the supply of machines and profits of the intermediate good producer :

$$x(v,t) = \left(\frac{\alpha^2 \phi(v,t)\, z(v,t)}{\chi + h(t)\, \phi(v,t)^{\frac{1}{\beta}}\, z(v,t)^{\frac{1}{\alpha\beta}}} \right)^{\frac{1}{1-\alpha}} L_Y(t) \tag{3.7}$$

By plugging the supply function of intermediate good producer (3.7) in price function (3.4) found in final good producer's program, we can express the profit and the price of intermediate good :

$$p(v,t) = \frac{\chi + h(t)\, \phi(v,t)^{\frac{1}{\beta}}\, z(v,t)^{\frac{1}{\alpha\beta}}}{\alpha} \tag{3.8}$$

and

$$\pi(v,t) = (1-\alpha)\, p(v,t)\, x(v,t) \tag{3.9}$$

By plugging equation (3.8) in (3.9) the profit of the intermediate good producer can be written as

$$\pi(v,t) = \frac{(1-\alpha)}{\alpha} \left(\chi + h(t)\, \phi(v,t)^{\frac{1}{\beta}}\, z(v,t)^{\frac{1}{\alpha\beta}} \right) x(v,t) \tag{3.10}$$

We can notice that profits are decreasing in the marginal cost of firm v : $m(v,t) = \chi + H(v,t)$ where $H(v,t) = h(t)\, \phi(v,t)^{\frac{1}{\beta}}\, z(v,t)^{\frac{1}{\alpha\beta}}$ represents the green tax burden. The green tax decreases the profits and its effect is heterogeneous across firms since final goods are differentiated in pollution intensity z i.e.

$$z(v,t) \neq z(i,t)\,,\ for\ v \neq i,\, h(t) > 0 \implies \pi(v,t) \neq \pi(i,t) \tag{3.11}$$

[8] We will discuss the effect of a pollution tax on R&D in details in further sections.

3.2.4 R&D Sector

In R&D sector, each laboratory aims at improving the labor productivity and also seeks to decrease the pollution intensity of intermediate goods. R&D innovations are modeled respecting a Poisson process with instantaneous arrival rate λL_R with $\lambda > 0$ which we can be interpreted as the creative destruction rate. Similar to Ricci (2007), to keep things simpler, we adopt only one type of R&D firm, which specializes in both productivity and pollution intensity improvements ϕ and z. However, the reader could consider this feature of modeling R&D unusual. A two-sector R&D model would require that expected profits should be the same to ensure that R&D activity in both sectors is maintained[9].

We can write the dynamics for implicit labor productivity and pollution intensity improvements ;

$$g_{\phi} = \frac{\dot{\bar{\phi}}_{max}\left(t\right)}{\bar{\phi}_{max}\left(t\right)} = \gamma_1 \lambda L_R, \quad \gamma_1 > 0 \tag{3.12}$$

$$g_Z = \frac{\dot{\underline{z}}_{min}\left(t\right)}{\underline{z}_{min}\left(t\right)} = \gamma_2 \lambda L_R, \quad \gamma_2 < 0 \tag{3.13}$$

where L_R is the labor allocated in R&D sector. A successful innovation allows the patent holder to provide the intermediate good with leading-edge technology $\bar{\phi}$ and the lowest pollution intensity $\underline{z}$. The parameter γ_2 shows the direction of the R&D activity. A negative γ_2 means that innovation is environmental friendly and its value shows at which extent innovation allows the production of cleaner intermediate goods. When $\gamma_2 = 0$, all goods have the same pollution intensity as in Nakada (2004). In this case, there is no differentiation of intermediate goods in terms of pollution intensity.

The free-entry condition ensures that arbitrage condition holds ;

$$w\left(t\right) = \lambda V\left(t\right) \tag{3.14}$$

[9] In case of asymmetric profits, there will be corner solutions where only one type of R&D will take place. Costa (2015) proposes a model with two R&D sectors and finds a balance of labor allocation between two R&D sectors which ensures the same expected value of R&D in both sectors. Recall that in his model, when the allocation of labor increases in one R&D sector, the other one sees its labor allocation increasing as well in order to avoid the corner solutions. This way of modeling two R&D sectors is surely more realistic but does not add different economic insights than the model with one R&D sector.

where $V(t)$ is the present value of expected profit streams. The equation (3.14) states that an agent is indifferent between working in the production sector and R&D sector. This ensures the equilibrium in the model at the balanced growth path. At equilibrium, when there is R&D activity, its marginal cost $w(t)$ is equal to its expected marginal value.

$$V(t) = \int_{\tau}^{\infty} e^{-\int_{\tau}^{t}(r(s)+\lambda L_R(s))ds} \pi\left(\bar{\phi}(t), \underline{z}(t)\right) dt \tag{3.15}$$

where $\pi\left(\bar{\phi}(t), \underline{z}(t)\right)$ denotes the profit at time t of a monopoly using the leading-edge technology available $\left(\bar{\phi}(t), \underline{z}(t)\right)$. r is the interest rate which is also the opportunity cost of savings and λL_R is the *creative destruction rate* of the economy. The creative destruction rate shows at which extent the incumbent firm is replaced by an entrant. Basically, it is the survival rate of the incumbent firm as an entrant makes the patent of incumbent firm obsolete.

Furthermore, the labor supply is fixed to unity and the market clearing condition is

$$L(t) = L_Y(t) + L_R(t) = 1 \tag{3.16}$$

The labor is allocated between final good production and R&D activity. Then, the cost of R&D activity is measured as a foregone final good production. The cost of producing the intermediate good enters in the resource constraint of the economy which is $Y(t) = c(t) + \chi x(t)$.

3.2.5 Household

We write the maximization program of household close to Tsur and Zemel (2009). The utility function of the household is

$$max E_T \left\{ \int_0^T u(c(t)) e^{-\rho t} dt + e^{-\rho T} \Gamma(a(T)) \right\} \tag{3.17}$$

where ρ is the pure time preference of household. $u\left(c\left(t\right)\right)$ is the utility coming from the consumption prior to an abrupt event which occurs at an uncertain date T. $\Gamma\left(a\left(t\right)\right)$ is the value function after the catastrophic event depending on the wealth accumulation $a\left(t\right)$. After integrating by parts the equation (3.17), the household's objective function reduces to

$$max\int_{0}^{\infty}u\left(c\left(t\right)+\bar{\theta}\Gamma\left(a\left(t\right)\right)\right)e^{-\left(\rho+\bar{\theta}\right)t}dt \tag{3.18}$$

where $\bar{\theta}$ is the constant probability of catastrophic event.

Discussion on the use of a constant hazard rate

Since our focus is on the balanced growth path analysis, the use of a constant hazard rate can be easily justified. To elaborate this, suppose that there is accumulation of the pollution stock and the hazard rate depends on this stock. In this case, it is easy to see that at balanced growth path, hazard would converge to a constant value.

In order to illustrate this, take the following hazard function (see Tsur and Zemel (2007)) $\theta\left(S\right)=\bar{\theta}-\left(\bar{\theta}-\underline{\theta}\right)e^{-bS}$ where S is the stock of pollution and $\underline{\theta}$ and $\bar{\theta}$ represent the lower and upper bound of the hazard rate respectively. It is easy to remark that $lim_{S\to\infty}=\theta\left(S\right)=\bar{\theta}$ and $lim_{S\to0}=\theta\left(S\right)=\underline{\theta}$.

Indeed, the use of the endogenous hazard rate matters only for transitional path but the endogenous probability converges to a constant hazard rate at the long run. In this chapter, we don't focus on transitional dynamics but make a balanced growth path analysis. Then, the purpose of the chapter justifies the use of a constant hazard rate.

Another interpretation regarding the use of constant hazard rate could be the following : abrupt events can be also triggered by flow pollution and not only by the stock pollution (see Bretschger and Vinogradova (2014)). Flow pollution highly affects the water, air and soil quality and consequently the agricultural activities. It can also cause consequent damage to the economy. For example, the sulfiric and nitric acid rain damage to man-made capital, buildings etc.

In addition to above mentioned explanations, a recent IPCC-Report (2014) claims that the frequency of tropical cyclones would remain unchanged.[10] Moreover, not every climate scientist agree on a variable climate induced changes in catastrophic event frequency. (See IPCC-Report (2014)).

What happens after the catastrophic event?

After the occurrence of an abrupt event, the economy is inflicted to a penalty which is proportional to the knowledge accumulation coming from R&D activity. Similar to Bréchet et al. (2012), the penalty function is $\psi(.)$ defined in the following manner :

A1. The penalty function $\psi(.) : \mathbb{R}_+ \to \mathbb{R}_+$ is twice continuously differentiable with following properties ; $\psi(a(t)) > 0$, $\psi_a(a(t)) < 0$, $\psi_{aa}(a(t)) > 0, \bar{\psi} > \psi(a(t))$.

$$\psi(a(t)) = \bar{\psi}(\omega - (1-\omega)\, log\,(a(t))) \tag{3.19}$$

where $\bar{\psi}$ is the amount of penalty. It is assumed that $0 < \omega < 1$. We argue that accumulating wealth helps an economy to better respond to negative consequences occurred due to catastrophic event. The empirical evidence suggests as well the higher capability of wealthier countries to adapt to climate change. (see Mendhelson et al. (2006)).

The parameter ω shows at which extent the knowledge accumulation can help the economy to respond against extreme events. The first term $\bar{\psi}\omega$ is the part of the damage that can not be recovered by the knowledge accumulation. The second expression $-\bar{\psi}(1-\omega)\, log\,(a(t))$ stands for the part of the damage that can be reduced by the wealth (knowledge) accumulation which takes place through R&D activity.

In order to ensure that the positive effect of wealth accumulation does not dominate the unrecoverable part of the damage, we make the assumption on the parameter ω (See Appendix 3.H for details about Assumption 2).

A2. $\omega > \frac{(\rho+\bar{\theta})ln(a(0))}{(\rho+\bar{\theta})(1+ln(a(0)))-g_Y}$

[10]In the future, it is likely that the frequency of tropical cyclones globally will either decrease or remain unchanged, but there will be a likely increase in global mean tropical cyclone precipitation rates and maximum wind speed. (IPCC-Report (2014), p.8)

The post value function as a function of the wealth can be given as

$$\Gamma\left(a\left(t\right)\right) = u\left(c_{min}\right) - \psi\left(a\left(t\right)\right) \tag{3.20}$$

where $u\left(c_{min}\right) = 0$ is the utility function where the consumption is reduced to the subsistence level. Note that the subsistence level consumption does not provide any utility (see Bommier et al. (2015)).

The household maximizes the objective function (3.17) subject to the following budget constraint

$$\dot{a}\left(t\right) = r\left(t\right)a\left(t\right) + w\left(t\right) - c\left(t\right) + T\left(t\right) \tag{3.21}$$

where $w\left(t\right)$ and $T\left(t\right)$ stand for wage and tax collected from the use of polluting intermediate good respectively. We make the assumption that government holds its budget balanced for $\forall t$, i.e., $h\left(t\right)P\left(t\right) = T\left(t\right)$. The solution of the dynamic optimization program should satisfy the no-Ponzi game condition $lim_{t\rightarrow\infty}e^{-\int_0^t r(s)ds}a\left(s\right) = 0$. Before deriving the Keynes-Ramsey rule from the maximization program of the household, it is crucial to show that the post value function depends on a stock variable[11] to ensure that the maximization problem is well posed.

Lemma 1. *$a\left(t\right) = V\left(t\right)$. Patents for innovations ($V\left(t\right)$ is the expected value of an innovation.) are held by households.*

Proof. See Appendix 3.C

From this lemma, we can remark that the wealth a of the household is proportional to the knowledge accumulation $\bar{\phi}_{max}$ and $\underline{z}_{min}$ which is a public good. Recall also that mitigation effort comes at the cost of lower adaptation since a lower pollution intensity z decreases the wealth accumulation. The lemma shows that the wealth is proportional to the expected value of the innovation which can be written by using equation (3.5) and the free-entry

[11]In our case, the physical constraint is the knowledge accumulation which stems from R&D activity.

condition (3.14)

$$a(t) = V(t) = \frac{w(t)}{\lambda} = \frac{(1-\alpha)}{\lambda}\frac{Y(t)}{L_Y(t)} = \frac{(1-\alpha)}{\lambda}\frac{\gamma_1 \alpha^{\frac{2\alpha}{1-\alpha}}}{1-\gamma_1}\left(\bar{\phi}_{max}(t)\,\underline{z}_{min}(t)\right)^{\frac{1}{1-\alpha}}\Omega_1(H) \tag{3.22}$$

where $Y(t) = \frac{\gamma_1}{1-\gamma_1}\alpha^{\frac{2\alpha}{1-\alpha}} L_Y \left(\bar{\phi}_{max}(t)\,\underline{z}_{min}(t)\right)^{\frac{1}{1-\alpha}}\Omega_1(H)$ is the aggregate production function[12] and $\Omega_1(H)$ is aggregation factor which is a function of the burden of the green tax H (see Appendix 3.G for aggregation factor). Indeed, the term $\Omega_1(H)$ stems from the aggregation of different firms indexed by $v \in [0,1]$. The green tax burden H is written

$$H(t) = \int_0^1 H(v,t)\,dv = h(t)\int_0^1 \phi(v,t)^{\frac{1}{\beta}} z(v,t)^{\frac{1}{\alpha\beta}}\,dv \tag{3.23}$$

The green tax burden $H(t)$ should be constant at the long run in order to ensure the existence of a balanced growth path. To do this, we should provide a policy rule (see Ricci (2007) and Nakada (2010)) that makes the green tax burden H constant at the long run. i.e, $\left(\frac{dH(t)}{dt} = 0\right)$.

$$g_h = -\left(\frac{g_Z}{\alpha\beta} + \frac{g_\phi}{\beta}\right) \tag{3.24}$$

g_i represents the growth rate of the variable i. According to the policy rule, the growth rate of the pollution tax $h(t)$ increases when the emission intensity decreases and it decreases while the total productivity increases. The policymaker makes this commitment which is credible since its aim is to keep the budget balanced. When pollution intensity decreases, revenues from tax collection decreases since aggregate pollution decreases. Contrary to this fact, when the total productivity increases, aggregate pollution increases as well as tax revenues. Then, the policy maker is able to decrease the growth rate of the pollution tax since its tax revenues increase. Different from Ricci (2007), the reason why the policy rule depends also on the productivity is due to our modeling the production function. Ricci (2007) takes the capital stock in his production function which is composed by the inter-

[12]Since there is an infinite number of firms in the economy, one should give an aggregate production function to make a balanced growth path analysis. See Appendix 3.A for the derivation of the aggregate production function.

mediate goods and productivity parameter. In our specification, the production function is only described by the intermediate goods $x(v,t)$.

Once the policy rule is established, it is possible to find the growth rate of the economy by differentiating equation (3.22),

$$g = g_Y = g_a = \frac{1}{1-\alpha}\left(g_\phi + g_Z\right) \tag{3.25}$$

The growth rate is positive if only $\gamma_1 > \gamma_2$.

3.3 Balanced Growth Path Analysis

In order to proceed to the balanced growth analysis, we start by solving the household's problem which is the maximization of the objective function (3.17) subject to the budget constraint (3.21). We assume a log utility function for household's utility as $u(c(t)) = log(c(t))$[13] for the analytical tractability of the model. By using the lemma 1, the Keynes-Ramsey rule is written[14]

$$g_c = \frac{\dot{c}(t)}{c(t)} = \left(r(t) - \left(\rho + \bar{\theta}\right) + \frac{\bar{\theta}\Gamma_V(V(t))}{\mu(t)}\right) \tag{3.26}$$

where $\mu(t)$ is the marginal utility of consumption per capita[15] (see Appendix 3.B). The growth rate of the consumption at the balanced growth rate is $g = g_Y = g_c$.

3.3.1 The Labor Allocation in Equilibrium

Once we have the Keynes-Ramsey equation, the labor allocation in R&D sector at the balanced growth path is (see Appendix 3.I for derivation)

[13]Note that we start our analysis with CRRA utility function $u(c(t)) = \frac{c^{1-\sigma}-1}{1-\sigma}$ where $c_{min} = 1$ and σ is the risk aversion parameter. This is the form of utility function when there is an abrupt event uncertainty. When the extreme climate event occurs, the consumption reduces to a subsistence level c_{min}. With this form of the utility function, it is easy to remark that $\lim\limits_{\sigma\to 1}\frac{c^{1-\sigma}-1}{1-\sigma} = log(c(t))$.

[14]In addition, we have $g = g_c = g_Y$ at the balanced growth path (see equation (A1.33)).

[15]Since $L(t) = 1$, we have $c(t) = C(t)$

$$L_R = \frac{\frac{\lambda\alpha^2(1-\omega)\bar{\theta}\bar{\psi}}{1-\alpha}\frac{\Omega_2(H)}{\Omega_1(H)} + \frac{\alpha\lambda\gamma_1}{(1-\gamma_1)}\frac{(\chi+H)^{-\frac{\alpha}{1-\alpha}}}{\Omega_1(H)} - \left(\rho+\bar{\theta}\right)}{\lambda + \frac{\lambda\alpha^2(1-\omega)\bar{\theta}\bar{\psi}}{1-\alpha}\frac{\Omega_2(H)}{\Omega_1(H)} + \frac{\alpha\lambda\gamma_1}{(1-\gamma_1)}\frac{(\chi+H)^{-\frac{\alpha}{1-\alpha}}}{\Omega_1(H)}} \tag{3.27}$$

where $\Omega_2(H)$ is the aggregation term for aggregate intermediate good $x(t)$. One can easily remark that the level of labor allocated in R&D sector depends on both catastrophic event probability, penalty rate and marginal cost of using a polluting intermediate good.

Proposition 1. *The market allocates much more labor to R&D with a higher catastrophe probability if the amount of penalty to due to catastrophic event is higher than a precised threshold*[16].

Proof. See Appendix 3.J

This result is counter-intuitive in the sense that catastrophic uncertainty is expected to decrease R&D activity since agents value the future less with a catastrophic event probability. It follows that with the discount effect, the interest rate for innovation patents increases as the impatience level of agents increases. To better understand the discount effect channel, we reformulate the interest rate which is constant at the balanced growth path (see Appendix 3.I for derivations).

$$r(t) = \frac{1}{1-\alpha}(g_\phi + g_Z) + \underbrace{\left(\rho+\bar{\theta}\right)}_{Discount\, effect} - \underbrace{\frac{\bar{\theta}\bar{\psi}(1-\omega)\alpha^2\Omega_2(H)}{\lambda(1-\alpha)\Omega_1(H)}(1-L_R)}_{Adaptation\, effect} \tag{3.28}$$

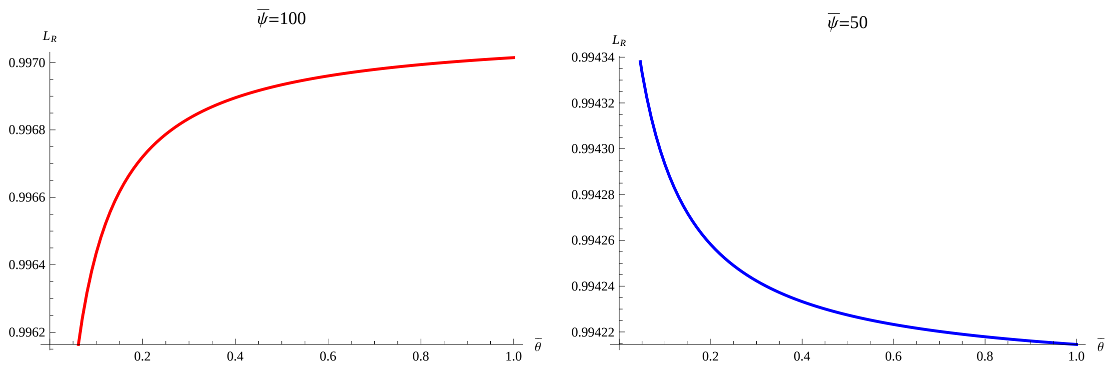
Contrary to standard Schumpeterian growth framework, the interest rate implies an additional term we call the adaptation effect. Since the economy becomes more resilient against abrupt events with wealth/knowledge accumulation, a higher abrupt event probability induces a higher marginal benefit from R&D patents. Then, the interest rate decreases through the adaptation effect. Consequently, the expected value of R&D increases with a lower interest rate (see equation (3.15)).

To sum up, it follows that there exist two opposite effects of abrupt event probability $\bar{\theta}$ on the interest rate which guides the investments in R&D activity. One may say that the adaptation effect dominates the discount effect if the penalty rate $\bar{\psi}$ due to the abrupt event

[16] The threshold is derived in Appendix 3.J.

exceeds a threshold. This relies on the fact that a higher penalty rate $\bar{\psi}$ implies a higher marginal benefit of R&D.

We illustrate the Proposition 1. graphically to confirm the mechanisms presented above by a numerical exercise.



(a) $\bar{\theta}$ increasing L_R

(b) $\bar{\theta}$ decreasing L_R

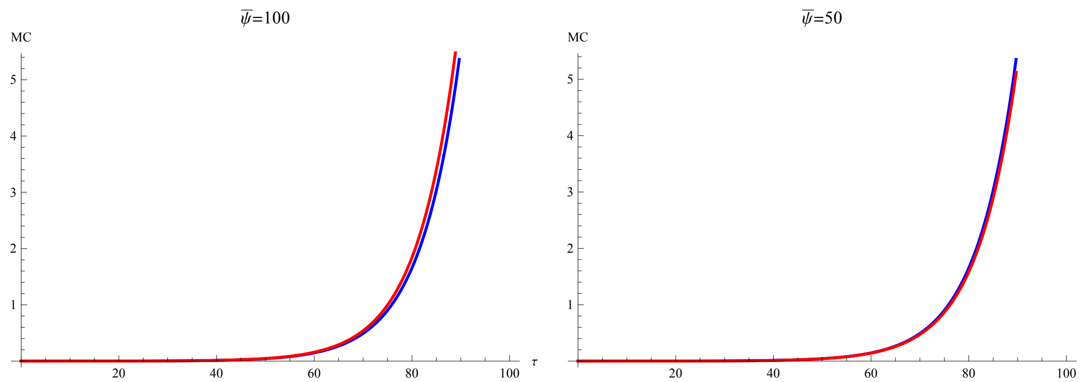
Figure 3.1: The effect of the abrupt event probability on labor allocation in R&D

How abrupt events and adaptation can create business opportunities and affect the competitiveness that promotes the long run growth rate in the market economy? To have an answer for this question, we should focus on the relationship between the labor allocation in R&D and the abrupt event probability. R&D activity changes the distribution of intermediate goods by skewing them towards cleaner ones. Then, the green tax burden becomes more stringent since the policy maker commits to follow an increasing path of pollution tax with cleaner intermediate goods. In order to understand this mechanism, we write the marginal cost of using the intermediate good in the following manner (see Appendix 3.E for the derivation)

$$m(\tau) = \chi + e^{g_h \tau} H \tag{3.29}$$

where τ stands for the age of an intermediate good. Recall that older vintages are dirtier than younger vintages. Indeed, the environmental policy rule by the policymaker creates a green crowding-out effect similar to Ricci (2007).

According to the figure (3.2a), the marginal cost of using the intermediate good increases when the abrupt event probability $\bar{\theta}$ increases the labor allocation in R&D. This is because the R&D activity increases and g_h becomes higher[17]. It follows that higher abrupt event probability $\bar{\theta}$ crowds out a higher number of old vintages which are dirtier from the market and replaces them by cleaner intermediate goods. Note that older vintages imply a higher green tax burden which decreases the competitiveness in the economy. Consequently, the abrupt event probability increases the competitiveness of the economy if the market shifts labor to R&D sector.



(a) $\bar{\theta}$ increasing L_R (b) $\bar{\theta}$ decreasing L_R

Figure 3.2: The effect of the abrupt event probability on the competitiveness of different vintages

However, a higher abrupt event probability can also allow a higher number of firms to stay on the market with dirty intermediate goods. This case is possible only if the abrupt event probability decreases the expected value of R&D. In the figure (3.2b), we observe that the marginal cost of using the intermediate good decreases with respect to the abrupt event probability $\bar{\theta}$ since the green tax burden becomes less stringent with a lower level of labor in R&D sector.

Proposition 2 . *(i) The effect of pollution tax is positive on growth if the elasticity of aggregation factor with respect to green tax burden H is high enough. (ii) This effect increases positively with catastrophic event probability if the amount of penalty is sufficiently high.*

[17]Equivalently, this means that environmental policy becomes more stringent.

Proof. See Appendix 3.K.

The economic explanation on the positive effect of pollution tax on growth is the following : the pollution tax decreases the demand of intermediate good since it becomes more costly to use polluting intermediate goods in the production as an input. It follows that the labor demand in the final good sector diminishes. As a result, the labor shifts from the final good sector to R&D sector which results in a higher creative destruction rate and hence more economic growth.

Moreover, one can understand this result more rigorously by looking at the elasticity of aggregation factors of production function $\Omega_1(H)$ and intermediate good demand $\Omega_2(H)$ with respect to green tax burden H. As expected, these terms are decreasing with green tax burden H. An important element that explains how pollution tax promotes the growth is the elasticity of these aggregation terms. We show that the elasticity of aggregation factor of production function is higher than the elasticity of aggregation factor of intermediate good factor. (see Appendix 3.L) This means that green tax affects negatively the final good sector more than the intermediate good sector. Equivalently, it means that the demand for intermediate good decreases less than the demand of final good. This results in a shift of labor from final good sector to R&D sector which aims to improve the productivity and emission intensity of intermediate goods. We also show that a necessary condition to have the positive effect of the pollution tax on growth is that the marginal cost of producing a machine $m = \chi + H$ is below a threshold.

In order to asses this effect more clearly, one may look at how labor allocation reacts to a change in marginal cost of pollution H. As R&D is known to promote growth in the economy when above mentioned conditions are fulfilled. The graphic shows the effect of the green tax burden H on labor allocation in R&D sector.

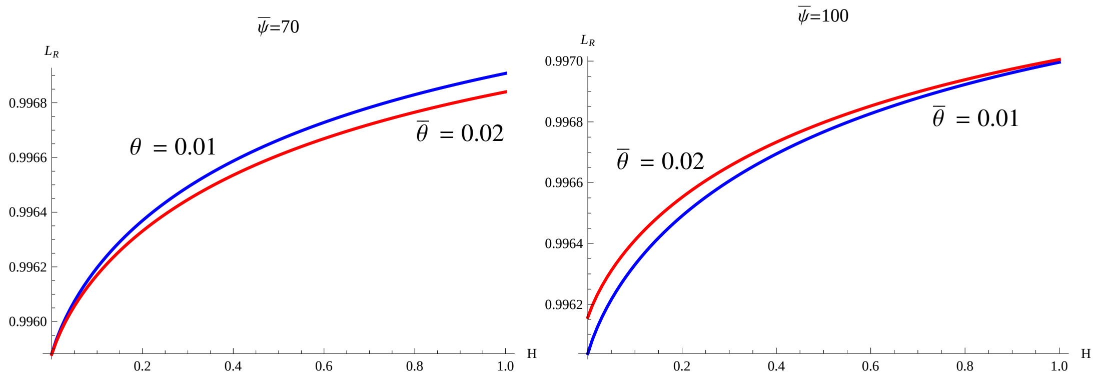


Figure 3.3: The effect of green tax burden H on labor allocation in R&D

An important remark is that R&D sector seeks to improve the productivity and emission intensity of intermediate goods. Then, the expected value of R&D is proportional to the profit of the monopolist intermediate good producer (see equation (3.15)). In this sense, we can argue that if the intermediate good demand decreases less than the final good demand, the labor is expected to shift from the final good sector to R&D sector.

It is worth discussing the relation between the abrupt event probability $\bar{\theta}$ and the effect of the pollution tax on growth. The positive effect of pollution tax on growth increases when the penalty rate $\bar{\psi}$ is above a precised threshold (see Appendix 3.J). This is due to the fact that the expected value of R&D increases since the interest rate decreases with a higher marginal benefit of R&D. Then, in the case where the adaptation effect dominates the discount effect, the positive effect of pollution tax on growth increases with a higher abrupt event probability $\bar{\theta}$.

3.4 Adaptation and Mitigation in a Market Economy

It is interesting to look at how the market economy adapts and mitigates when it faces a higher catastrophe event probability $\bar{\theta}$. To assess the implications of the pollution tax on adaptation of the economy, one should observe how the value of R&D $V(t)$ changes with respect to catastrophic event probability. Recall that knowledge accumulation that allows the adaptation stems from R&D activity. An economy that accumulates knowledge becomes wealthier (see lemma 1). On the other hand, the mitigation activity can be captured through variable Z, which stands for the pollution intensity.

Indeed, it is worthwhile to note that the market economy does not target explicitly to do adaptation and mitigation activities. It is clear in our framework that adaptation and mitigation activities are promoted by the means of R&D activity which aims primarily to have R&D patents that provide dividends to shareholders. Then, it is plausible to say that adaptation and mitigation mix are the natural outcome of the R&D in the market. A proxy indicator can be easily constructed to understand how adaptation and mitigation balance is found in the market economy.

The variable $M = \frac{1}{Z}$ can be considered as the mitigation activity. As the pollution intensity decreases, mitigation increases. The economy starts to adapt more when the knowledge stock increases. This means that when wealth accumulation a increases, the resilience against a climatic catastrophe increases. The growth rate of adaptation and mitigation is given by

$$g_A = \frac{1}{1-\alpha}\left(\gamma_1 + \gamma_2\right)\lambda L_R$$

$$g_M = -\gamma_2 \lambda L_R$$

$$g_{\frac{A}{M}} = \left(\frac{\gamma_1}{1-\alpha} + \left(1 + \frac{1}{1-\alpha}\right)\gamma_2\right)\lambda L_R$$

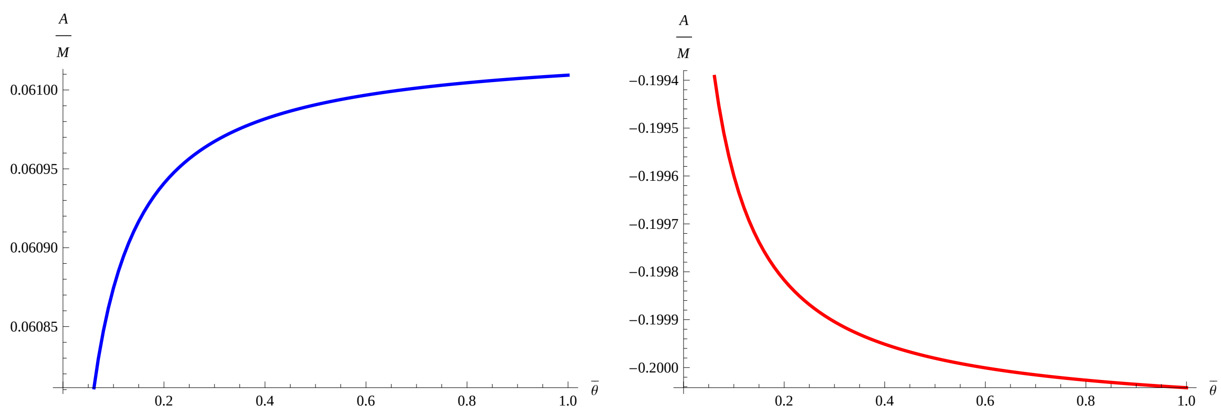
Proposition 3. *(i) At the balanced growth path, the growth rate of adaptation is higher than that of mitigation if the cleanliness rate of R&D is not sufficiently high (γ_2).*

$$Case\,1.\, g_{\frac{A}{M}} > 0\, if\; -\left(\frac{\gamma_1}{\gamma_2}\right) > 2-\alpha$$

$$Case\,2.\, g_{\frac{A}{M}} < 0\, if\; -\left(\frac{\gamma_1}{\gamma_2}\right) < 2-\alpha$$

In case 1, when the cleanliness rate of R&D γ_2 is not high enough relative to the total productivity γ_1, the growth rate for adaptation/mitigation ratio $\frac{A}{M}$ is positive. Then, the economy adapts always much more than it mitigates at the long run. In case 2, the economy offers cleaner innovations compared to the case 1. Therefore, the growth rate of adaptation/mitigation ratio is negative, which means that mitigation is higher than adaptation. It is interesting to focus on the relation between the catastrophic event probability $\bar{\theta}$ and the equilibrium level of adaptation and mitigation. Taking into consideration proposition 1, when the economy facing a high-level penalty rate allocates more labor to R&D activities, the growth rate of adaptation is higher than the that of mitigation in case 1 and vice versa in case 2.

We illustrate this result numerically :



(a) $g_{\frac{A}{M}} > 0$ with $\left(\frac{\gamma_1}{\gamma_2}\right) > 2 - \alpha$ (b) $g_{\frac{A}{M}} < 0$ with $\left(\frac{\gamma_1}{\gamma_2}\right) < 2 - \alpha$

Figure 3.1: Growth rate of adaptation/mitigation

As one can see, the economy starts to accumulate more wealth with higher catastrophe probability $\bar{\theta}$ in order to adapt to the penalty due to the catastrophic event. In case where the penalty rate is not high, economy would allocate less labor to R&D. Then, ratio adaptation/mitigation would fall, which means that the growth rate of mitigation becomes higher relative to that of adaptation when the economy faces a higher risk of abrupt event.

In the figure (3.1b), the ratio of total productivity and cleanliness of R&D $\left(\frac{\gamma_1}{\gamma_2}\right)$ is low. Therefore, the market mitigates more than it adapts to the catastrophic event. Moreover, we remark an interesting result linked with adaptation and mitigation activities. When the cleanliness of R&D is higher, the economic growth decreases since the R&D offers cleaner intermediate goods that are less productive (see Ricci (2007), (see Aghion and Howitt (1997)[18]). This leads to a decrease in final good production Y. Then, it follows that the growth rate of mitigation comes at the cost of the growth rate of adaptation.

A similar result is also present in Tsur and Zemel (2016a) and Bréchet et al. (2012) but the difference is that in our model, the growth rate of adaptation and mitigation is always positive in the market economy. Consequently, the economy always increases its adaptation and mitigation level at each date. However, in Tsur and Zemel (2016a), Bréchet et al. (2012) and Kama and Pommeret (2016), the trade-off relies on the optimal allocation of resources between adaptation and mitigation. It follows that when the economy invests more in adaptation, this comes at the cost of mitigation investments. Nonetheless, when adaptation and mitigation activities come as a natural outcome from the R&D sector and both of them grow at the long run, so we are not allowed to mention a trade-off between adaptation and mitigation in our framework.

The growth rate of adaptation, mitigation and pollution at the long run is

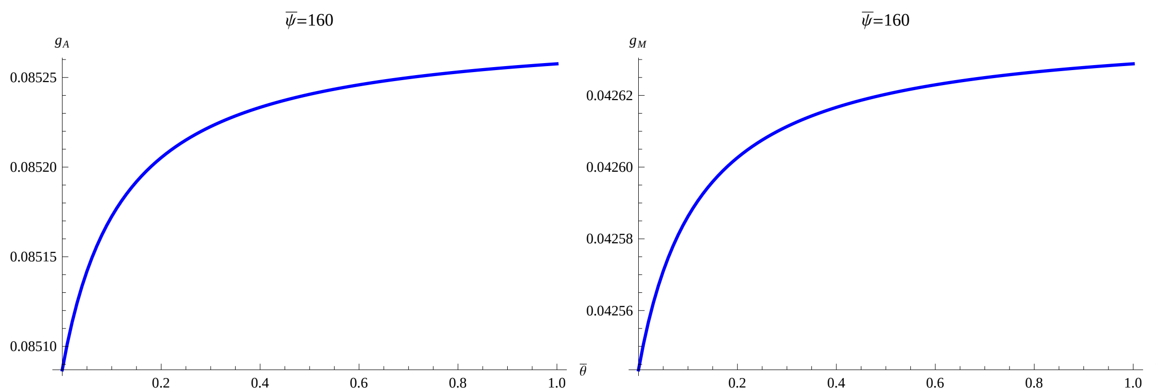


Figure 3.2: Growth rate of adaptation and mitigation

Keeping in mind that the economy grows and adapts to abrupt events at each date, one may ask how the aggregate pollution evolves at the long run. Despite the relaxation of the trade-off between adaptation and mitigation in a decentralized economy, we show that a new trade-off between adaptation and pollution arises in the market economy.

[18]The authors argue that capital intensive intermediate goods are more productive.

Before presenting the trade-off between adaptation and pollution, we write the aggregate pollution

$$P(t) = \left[\bar{\phi}_{max}(t)\right] \left[\underline{z}_{min}(t)\right]^{\frac{1}{\alpha\beta}} Y(t) \tag{3.30}$$

It is easy to remark that pollution $P(t)$ is proportional to aggregate production $Y(t)$. Differentiating equation (3.30), at the long run, pollution growth can be written

$$g_P = \left(\frac{2-\alpha}{1-\alpha} g_\phi + \frac{1+\frac{(1-\alpha)}{\alpha\beta}}{1-\alpha} g_Z\right) = \frac{1}{1-\alpha}\left((2-\alpha)\gamma_1 + \left(1+\frac{(1-\alpha)}{\alpha\beta}\right)\gamma_2\right)\lambda L_R \tag{3.31}$$

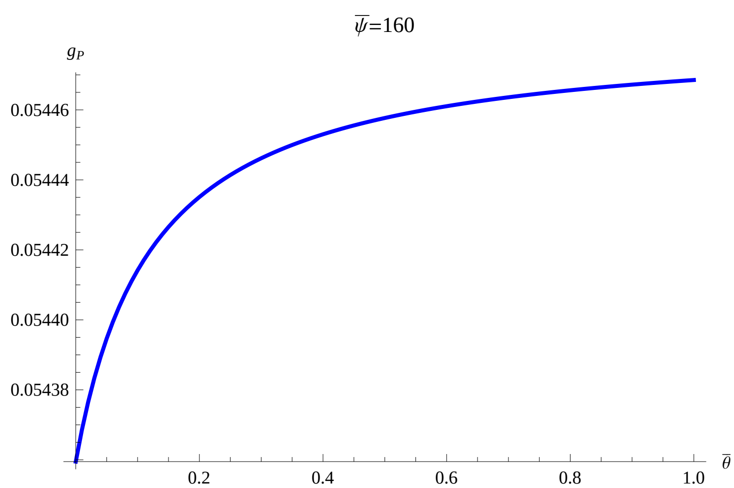


Figure 3.3: Growth rate of pollution

The numerical exercise confirms that when the economy adapts to abrupt event when it faces a higher abrupt event probability, the pollution growth is higher as well despite the higher growth rate of mitigation at the long run. This outcome is due to scale effect mentioned above. In fact, this result challenges the adaptation and mitigation trade-off but reveals a new trade-off between adaptation and pollution.

Proposition 4. *Pollution growth at the balanced growth path depends on the pollution share*

$\alpha\beta$, the cleanliness of R&D γ_2 and the total productivity parameter γ_1. In case of higher labor allocation in R&D with abrupt event probability $\bar{\theta}$ and green tax burden H, the growth rate of pollution is positive if the pollution share or cleanliness of R&D are not sufficiently high. In this case, economy faces a Jevons type paradox.

$$Case\, 1.\, g_P > 0\, if\, -\left(\frac{\gamma_1}{\gamma_2}\right) > \frac{\left(1+\frac{(1-\alpha)}{\alpha\beta}\right)}{2-\alpha} \tag{3.32}$$

$$Case\, 2.\, g_P < 0\, if\, -\left(\frac{\gamma_1}{\gamma_2}\right) < \frac{\left(1+\frac{(1-\alpha)}{\alpha\beta}\right)}{2-\alpha} \tag{3.33}$$

In the market economy, pollution can grow, albeit the presence of cleaner intermediate goods, when the economy allocates much more labor to R&D. Indeed, total productivity improvements with R&D activity increases the scale of the economy. Due to the scale effect, pollution growth at the long run turns out to be higher if R&D does not offer sufficiently cleaner intermediate goods. This result can be referred to as Jevons Paradox which claims that technological improvements increases the efficiency of energy used in the production but also increases the demand of energy. In the Schumpeterian economy, the intermediate godo demand increases with the scale effect. Consequently, the pollution growth can be higher even with cleaner intermediate goods.

An illustrative example about this topic could be India's increased aggregate pollution despite the reduction of the pollution intensity. Ollivier and Barrows (2016) shows that pollution intensity decreases in India between 1990-2010. However, the emissions have increased in India between this period[19].

3.5 Welfare Analysis

Once, we show that in a Schumpeterian economy, a new trade-off arises between adaptation and pollution, it is desirable to study the welfare implications regarding this new trade-off

[19]See World Bank Database : http://data.worldbank.org/indicator/EN.ATM.CO2E.PC?end=2010&locations=IN&start=1990

with respect to the abrupt event probability $\bar{\theta}$ and green tax burden H at the balanced growth path. We know that with adaptation, pollution growth can be higher if the condition (3.32) is ensured. Then, how the welfare of the household is affected when adaptation (wealth accumulation) increases? Using equation (3.20), the total welfare can be described as

$$W^* = \int_0^\infty \left[log\left(c\left(t\right)\right) + \bar{\theta}\left(u\left(c_{min}\right) - \bar{\psi}\left(\omega - \left(1-\omega\right) log\left(a\left(t\right)\right)\right)\right)\right] e^{-\left(\rho+\bar{\theta}\right)t} dt \tag{3.34}$$

By integrating the welfare function (3.34) and using the lemma 1, we have

$$W^* = \frac{log\left(\frac{\alpha^2\Omega_2(H)}{\Omega_1(H)}Y\left(0\right)\right) - \bar{\psi}\bar{\theta}\left(\omega + \left(1-\omega\right) log\left(\frac{\gamma_1(1-\alpha)\alpha^{\frac{2}{1-\alpha}}\phi_{max}(0)\underline{z}_{min}(0)}{1-\gamma_1}\right)\right)}{\rho + \bar{\theta} - g} \tag{3.35}$$

A higher abrupt event probability and green tax burden have two opposite effects on the welfare of the household. The first effect is the output effect. When the abrupt event probability increases the labor allocation in R&D sector, the final good production decreases as well as the consumption. Consequently, the welfare decreases with the output effect. On the other hand, since the labor allocation in R&D sector increases, the growth rate of the economy at the long run increases. This can be called the growth effect. If the growth effect is higher than the output effect, the welfare increases (see Appendix 3.L).

Proposition 5 . *The welfare is affected negatively with respect to green tax burden and abrupt event probability if they don't enhance the growth rate of the economy. This result depends on the total productivity γ_1 and cleanliness of R&D γ_2.*

Proof. See Appendix 3.L.

In the numerical exercise, the welfare is shown to be decreasing with respect to green tax burden H and abrupt event probability $\bar{\theta}$ when the cleanliness rate of R&D is close to the total productivity of R&D activity[20]. To sum up, the abrupt event probability and green tax

[20] $\rho = 0.05$, $\omega = 0.1$, $\bar{\psi} = 160$, $\alpha = 0.42$, $\beta = 0.06$, $\lambda = 0.001$, $\chi = 1$. In addition to these parameter values, the welfare with respect to $\bar{\theta}$ and H increases when $\gamma_1 = 0.75$ and $\gamma_2 = -0.25$ and decreases when $\gamma_1 = 0.95$ and $\gamma_2 = -0.9$

burden do not increase the growth rate of the economy and the welfare is always negatively affected.

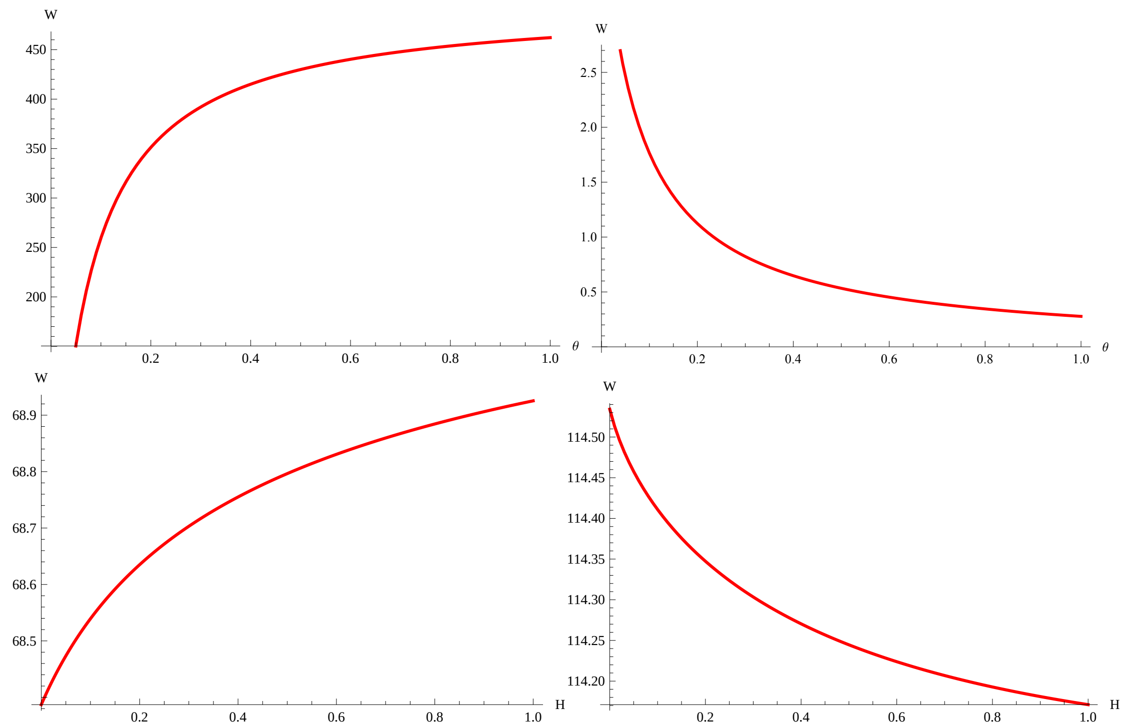


Figure 3.1: The effect of abrupt event probability $\bar{\theta}$ and green tax burden H on welfare

This result is plausible because when the cleanliness rate of R&D γ_2 is high, the intermediate goods are replaced by cleaner ones. In this case, two important thing happens ; first, the output decreases since cleaner intermediate goods are less productive. Second, the growth rate of the economy decreases with cleaner intermediate goods since the burden of green tax increases with cleaner intermediate goods. Whereas, the welfare increases with abrupt event probability and green tax burden if both of them increases the growth rate of the economy. In this case, there is a reduction of output since the labor shifts from final good sector to R&D sector. However, if the total productivity is sufficiently high, the growth effect can compensate the output effect and the welfare can increase.

3.6 Conclusion

In this chapter, our contribution builds on the analysis of adaptation and mitigation through an endogenous R&D process in a decentralized economy. The existing literature treated the adaptation and mitigation policy mix in the social optimum framework without taking into account the presence of an endogenous R&D decision making.

We examine the effect of catastrophe probability on R&D decisions of the market economy. R&D activity aims to improve the total productivity of labor and the emission intensity of intermediate goods. Additionally, R&D serves to adapt to damage from abrupt events as well. We show that higher abrupt event probability increases the R&D if the penalty rate is above a threshold. This result relies on the fact that marginal benefit of R&D increases since innovation patents helps to decrease the vulnerability against abrupt event damage.

Similar to Hart (2004) and Ricci (2007), we show that pollution tax can promote the growth rate of the economy. Differently from these studies, the effect of pollution tax with respect to abrupt event probability is shown to be higher or lower depending on the penalty rate.

The market economy starts to accumulate more knowledge and to adapt more if the total productivity of R&D is higher than the cleanliness of innovations. This fact relies on the assumption that cleaner intermediate goods are less productive. Then, the growth rate turns out to be lower at the long run. This means that mitigation comes at the cost of wealth accumulation at the long run. However, in a growing economy at the long run, the trade-off between adaptation and mitigation is not relevant as much as claimed in many studies (see Tsur and Zemel (2016a), Zemel (2015)) since adaptation and mitigation are both shown to grow at the long run. We show that a new trade-off between adaptation and pollution can arise. Since the wealth accumulation (adaptation) increases the growth rate of the economy at the long run, the pollution growth can be higher due to the increased scale of the economy. This result shows the possibility of a Jevons paradox since the economy emits more pollution with cleaner intermediate goods.

Lastly, we analyze the implications of the abrupt event probability and green tax burden on welfare. We show that there exists two opposite effects which are the output and growth effect. When the green tax boosts the economy, there is a shift of labor from final good sector to R&D sector which decreases the output. However, we show that this negative effect on output and hence on welfare can be compensated by the growth effect and the welfare can be higher with a higher abrupt event probability and green tax burden.

Appendix

3.A Production Function

As in Ricci (2007), we define the function as

$$Y(t) = \int_0^1 \left(\phi(v,t)\, L_Y(t)\right)^{1-\alpha} \left(P(v,t)^{\beta}\, x(v,t)^{1-\beta}\right)^{\alpha} dv \tag{A1.1}$$

where $P(v,t)$ is the polluting input. From production function, we can define a emissions-intermediate good ratio in order to have simpler form for production function ;

$$z(v,t) = \left(\frac{P(v,t)}{\phi(v,t)^{\frac{1}{\beta}}\, x(v,t)}\right)^{\alpha\beta} \tag{A1.2}$$

The production function takes a simpler form

$$Y(t) = L_Y(t)^{1-\alpha} \int_0^1 \phi(v,t)\, z(v,t)\, x(v,t)^{\alpha}\, dv \tag{A1.3}$$

3.B Household's Maximization Program

The Hamiltonian for the maximization program reads

$$\mathcal{H} = u(c(t)) + \bar{\theta}\Gamma(a(t)) + \mu\left(r(t)\, a(t) + w(t) - c(t) + T(t)\right) \tag{A1.4}$$

The first-order conditions can be written

$$u_c(c) = \mu \tag{A1.5}$$

$$\frac{\dot{\mu}}{\mu} = \left(\rho + \bar{\theta}\right) - r - \frac{\bar{\theta}\Gamma\left(a\left(t\right)\right)}{\mu} \tag{A1.6}$$

With $u(c) = log(c)$. The Keynes-Ramsey equation yields

$$\frac{\dot{c}}{c} = \left(r - \left(\rho + \bar{\theta}\right)\right) + \frac{\bar{\theta}\Gamma_a(a)}{u_c(c)} \tag{A1.7}$$

By making trivial algebra, we can reformulate equation (A1.7) as (3.26).

3.C Proof of Lemma 1

We can reformulate the budget constraint in the form

$$\dot{a}(t) = r(t)\,a(t) + w(t) - c(t) + T(t) \tag{A1.8}$$

With the perfect competition assumption in final good sector, the profits are equal to zero.

$$c(t) + \chi x(t) = Y(t) = w(t)\,L_Y(t) + \int_0^1 p(v,t)\,x(v,t) \tag{A1.9}$$

By replacing zero profit condition (A1.9) in budget constraint of the household (A1.8), the budget constraint becomes

$$\dot{a}(t) = r(t)\,a(t) + w(t)\,L_R(t) - \left[\int_0^1 p(v,t)\,x(v,t) - h(t)\,P(t) - \chi x(t)\right] \tag{A1.10}$$

From free-entry condition in R&D sector, we know $\lambda L_R(t)\,V(t) - w(t)\,L_R(t) = 0$. Recall that the term in brackets is the total profit $\pi(t) = \int_0^1 \pi(v,t)$ in intermediate good sector. Then, the budget constraint becomes

$$\dot{a}(t) = r(t)\,a(t) + \lambda L_R(t)\,V(t) - \pi(t) \tag{A1.11}$$

Consequently, the Hamilton-Jacobi-Bellman equation for expressing the expected value of an innovation in R&D sector allows us to conclude that

$$a(t) = V(t) \tag{A1.12}$$

This completes the proof of Lemma 1.

3.D Cross-Sectoral Distribution

Productivity Distribution

We follow a method similar to Aghion and Howitt (1997) in order to characterize long-run distribution of relative productivity terms, both for technology improvements $\phi(v,t)$ and emission intensity $z(v,t)$. Let $F(.,t)$ be the cumulative distribution of technology index ϕ across different sectors at a given date t and write $\Phi(t) \equiv F(\phi,t)$. Then

$$\Phi(0) = 1 \tag{A1.13}$$

$$\frac{\dot{\Phi}(t)}{\Phi(t)} = -\lambda L_R(t) \tag{A1.14}$$

Integrating this equation yields

$$\Phi(t) = \Phi(0)\,e^{-\lambda\gamma_1 \int_0^t L_R(s)ds} \tag{A1.15}$$

The equation (A1.13) holds because it is nossible t possible that a firm has a productivity parameter ϕ larger than the leading firm in the sector. The equation (A1.14) means that at each date a mass of λn firm lacks behind, due to innovations that take place with Poisson distribution. From equation (3.12), we write

$$\frac{\dot{\bar{\phi}}_{max}\left(t\right)}{\bar{\phi}_{max}\left(t\right)}=\gamma_1\lambda L_R \tag{A1.16}$$

Integrating equation (A1.16), we have ;

$$\bar{\phi}_{max}\left(t\right)=\bar{\phi}_{max}\left(0\right)e^{\lambda\gamma_1\int_0^t L_R(s)ds} \tag{A1.17}$$

where $\bar{\phi}_{max}\left(0\right)\equiv\bar{\phi}$. By using equations (A1.15) and (A1.17), we write

$$\left(\frac{\bar{\phi}}{\bar{\phi}_{max}}\right)^{\frac{1}{\gamma_1}}=e^{-\lambda\int_0^t L_R(s)ds}=\Phi\left(t\right) \tag{A1.18}$$

We define a to be the relative productivity $\frac{\bar{\phi}}{\bar{\phi}_{max}}$. Basically, $\Phi\left(t\right)$ is the probability density distribution.

Emission Intensity Distribution

By proceeding exactly in same manner, we have

$$\frac{\dot{\underline{z}}_{min}\left(t\right)}{\underline{z}_{min}\left(t\right)}=\gamma_2\lambda L_R \tag{A1.19}$$

By integrating equation (A1.19), we have

$$\underline{z}_{min}\left(t\right)=\underline{z}_{min}\left(0\right)e^{\lambda\gamma_2\int_0^t L_R(s)ds} \tag{A1.20}$$

We rewrite the equation as

$$\left(\frac{\underline{z}}{\underline{z}_{min}}\right)^{\frac{1}{\gamma_2}} = e^{-\lambda \int_0^t L_R(s)ds} \tag{A1.21}$$

We can easily remark that this last equation is the same that we have found in equation (A1.18). We write

$$\left(\frac{\bar{\phi}}{\bar{\phi}_{max}}\right)^{\frac{1}{\gamma_1}} = \left(\frac{\underline{z}}{\underline{z}_{min}}\right)^{\frac{1}{\gamma_2}} \tag{A1.22}$$

From equation (A1.22), We can find the relative distribution for emission intensity across firms

$$\frac{\underline{z}}{\underline{z}_{min}} = \left(\frac{1}{a}\right)^{-\frac{\gamma_2}{\gamma_1}}$$

3.E Marginal Cost of Using Intermediate Good

We know that the marginal cost of using a given machine v is the following

$$m\left(v,t\right) = \chi + H\left(v,t\right) \tag{A1.23}$$

where $H\left(v,t\right) = h\left(t\right)\phi\left(v,t\right)z\left(v,t\right)^{\frac{1}{\alpha\beta}}$. It is possible to represent equations (A1.21) and (A1.22) in terms of their vintage v,

$$\left(\frac{\bar{\phi}_{max}\left(t-v\right)}{\bar{\phi}_{max}\left(v\right)}\right)^{\frac{1}{\gamma_1}} = e^{-\lambda \int_0^v L_R(s)ds} \tag{A1.24}$$

$$\left(\frac{\underline{z}_{min}\left(t-v\right)}{\underline{z}_{min}\left(v\right)}\right)^{\frac{1}{\gamma_2}} = e^{-\lambda \int_0^v L_R(s)ds} \tag{A1.25}$$

Using equations (A1.24) and (A1.25), we find the equation

$$m(v) = \chi + e^{\left(\frac{g_Z}{\alpha\beta} + g_\phi\right)v} H \tag{A1.26}$$

3.F Aggregate Economy

We replace equation of supply of machines (3.7) in equation (3.1) and write

$$Y(t) = L_Y(t) \int_0^1 \phi(v,t)\, z(v,t) \left(\frac{\alpha^2 \phi(v,t)\, z(v,t)}{\chi + h(t)\, \phi(v,t)^{\frac{1}{\beta}}\, z(v,t)^{\frac{1}{\alpha\beta}}} \right)^{\frac{\alpha}{1-\alpha}} dv \tag{A1.27}$$

We proceed to reformulate the production in a way that it is possible to write productivity and emission intensity gaps. Note that according to Aghion and Howitt (1997), they are constant along time. By dividing and multiplying nominator and denominator by $\bar{\phi}_{max}\, \underline{z}_{min}$;

$$Y(t) = \alpha^{\frac{2\alpha}{1-\alpha}} L_Y \left(\bar{\phi}_{max}\, \underline{z}_{min}\right)^{\frac{1}{1-\alpha}} \int_0^1 \left[\left(\frac{\phi(v,t)}{\bar{\phi}_{max}} \frac{z(v,t)}{\underline{z}_{min}} \right)^{\frac{1}{1-\alpha}} \left(\frac{1}{\left(\chi + h(t)\, \bar{\phi}_{max}^{\frac{1}{\beta}}\, \underline{z}_{min}^{\eta} \left(\frac{z(v,t)}{\underline{z}_{min}} \right)^{\frac{1}{\alpha\beta}} \frac{\phi(v,t)}{\bar{\phi}_{max}} \right)} \right)^{\frac{\alpha}{1-\alpha}} dv \right] \tag{A1.28}$$

By using the productivity and emission intensity distributions, We find the aggregate production function as follows ;

$$Y(t) = \frac{\gamma_1}{1-\gamma_1} \alpha^{\frac{2\alpha}{1-\alpha}} L_Y \left(\bar{\phi}_{max}(t)\, \underline{z}_{min}(t)\right)^{\frac{1}{1-\alpha}} \Omega_1(H) \tag{A1.29}$$

where the aggregation function for production $\Omega_1(H)$;

$$\Omega_1(H) = \int_0^1 \frac{a^{\frac{1}{1-\alpha}\left(1+\frac{\gamma_2}{\gamma_1}\right)}}{\left(1+\frac{H}{\chi}a^{\frac{1}{\beta}+\frac{\gamma_2}{\gamma_1}\frac{1}{\alpha\beta}}\right)^{\frac{\alpha}{1-\alpha}}}\nu'(a)\,da \tag{A1.30}$$

where $H = h(t)\,\bar{\phi}_{max}\underline{z}_{min}^{\eta}$ which is a constant term along time t by the policy rule and $\nu'(a)$ is the density function for the function $\nu(a) = F(.,t) = a^{\frac{1}{\gamma_1}}$.

The aggregation of intermediate factor $x(t)$ is obtained in same manner.

$$x(t) = \int_0^1 x(v,t)\,dv = \frac{\gamma_1}{1-\gamma_1}\alpha^{\frac{2}{1-\alpha}}L_Y\left(\bar{\phi}_{max}(t)\,\underline{z}_{min}(t)\right)^{\frac{1}{1-\alpha}}\Omega_2(H) \tag{A1.31}$$

where the aggregation factor $\Omega_2(H)$ for intermediate good $x(t)$ is

$$\Omega_2(H) = \int_0^1 \frac{a^{\frac{1}{1-\alpha}\left(1+\frac{\gamma_2}{\gamma_1}\right)}}{\left(1+\frac{H}{\chi}a^{\frac{1}{\beta}+\frac{\gamma_2}{\gamma_1}\frac{1}{\alpha\beta}}\right)^{\frac{1}{1-\alpha}}}\nu'(a)\,da \tag{A1.32}$$

The final good market equilibrium yields $Y(t) = c(t) + \chi x(t)$, since some part of the final good is used for the production of intermediate good. From equation (3.6), we know that aggregate cost of the production good $x(t)$is given by $\chi x(t)$.

$$c(t) = Y(t) - \chi x(t) = \alpha^2\frac{\Omega_2(H)}{\Omega_1(H)}Y(t) \tag{A1.33}$$

which gives the consumption $c(t)$ as a function of production function $Y(t)$.

3.G Aggregation Factor

From production function, in order to solve the integral (A1.30),

$$\Omega_1(H) = \int_0^1 \frac{a^{\bar{\gamma}}}{\left(1+\frac{H}{\chi}a^{\frac{1}{\beta}+\frac{\gamma_2}{\gamma_1}}\right)^{\frac{\alpha}{1-\alpha}}}da \tag{A1.34}$$

where $\bar{\gamma} = \frac{1}{1-\alpha}\left(1+\frac{\gamma_2}{\gamma_1}\right)+\frac{1}{\gamma_1}-1$. We use the substitution method. We define

$$y = -\frac{H}{\chi}a^{\frac{1}{\beta}+\frac{\gamma_2}{\gamma_1}\frac{1}{\alpha\beta}} \quad and \quad dy = -\left(1+\frac{\gamma_2}{\gamma_1}\right)\frac{H}{\chi}a^{\frac{1}{\beta}+\frac{\gamma_2}{\gamma_1}\frac{1}{\alpha\beta}-1}da \tag{A1.35}$$

We rewrite the aggregation factor,

$$\Omega_1\left(H\right) = -\int_{-\frac{H}{\chi}}^{0} y^{\frac{\bar{\gamma}+1-b}{b}}\left(1-y\right)^{-\frac{\alpha}{1-\alpha}}dy \tag{A1.36}$$

where $b = \frac{1}{\beta}+\frac{\gamma_2}{\gamma_1}\frac{1}{\alpha\beta}$. It is easy to remark that expression in the integral is the incomplete beta function. Then, we can express this integral by using Gaussian hypergeometric function as follows

$$\Omega_1\left(H\right) = \left(\frac{1}{1+\bar{\gamma}}\right){}_2F_1\left(\frac{\bar{\gamma}+1}{b},\frac{\alpha}{1-\alpha};\frac{\bar{\gamma}+b+1}{b};-\frac{H}{\chi}\right) \tag{A1.37}$$

In order to see the marginal change of aggregation factor with respect to marginal cost of pollution H ;

$$\frac{\partial\Omega_1\left(H\right)}{\partial H} = -\frac{1}{\chi}\left(\frac{\alpha\left(\bar{\gamma}+1\right)}{\left(1-\alpha\right)\left(\bar{\gamma}+1+b\right)}\right){}_2F_1\left(\frac{\bar{\gamma}+1}{b}+1,\frac{\alpha}{1-\alpha}+1;\frac{\bar{\gamma}+b+1}{b}+1;-\frac{H}{\chi}\right)<0$$

3.H Condition on Penalty Function

From the household problem, we define the post-value function as

$$\Gamma\left(a\left(t\right)\right) = u\left(c_{min}\right)-\psi\left(a\left(t\right)\right) \tag{A1.38}$$

and the penalty function

$$\psi\left(a\left(t\right)\right)=\bar{\psi}\left(\omega-\left(1-\omega\right)log\left(a\left(t\right)\right)\right) \tag{A1.39}$$

At Balanced Growth Path, the post value function can be written in the following manner ;

$$\Gamma^{*}=-\int_{0}^{\infty}\psi\left(a\left(t\right)\right)e^{-\left(\rho+\bar{\theta}\right)t}dt=-\bar{\psi}\left(\frac{\omega}{\rho+\bar{\theta}}-\frac{\left(1-\omega\right)log\left(a\left(0\right)\right)}{\rho+\bar{\theta}-g_{Y}}\right) \tag{A1.40}$$

and

$$\omega>\frac{\left(\rho+\bar{\theta}\right)ln\left(a\left(0\right)\right)}{\left(\rho+\bar{\theta}\right)\left(1+ln\left(a\left(0\right)\right)\right)-g_{Y}} \tag{A1.41}$$

where $a\left(0\right)$ is the level of wealth at initial date.

3.I Labor Allocation in Equilibrium

To find the labor allocation in R&D sector, we differentiate equation (3.15) that yields the Hamilton-Jacobi-Bellman equation at the balanced growth path

$$\left(r+\lambda L_{R}\right)-\frac{\dot{V}\left(t\right)}{V\left(t\right)}=\frac{\pi\left(\bar{\phi}_{max},\underline{z}_{min}\right)}{V\left(t\right)} \tag{A1.42}$$

The lemma 1 shows that household owns the firms in market. Household receives dividend from innovation assets on the market.

With the functional forms defined in the text and using the resource constraint $Y\left(t\right)=c\left(t\right)+x\left(t\right)$. The growth rate of economy can be written

$$g_{c}=\frac{\dot{c}\left(t\right)}{c\left(t\right)}=r\left(t\right)-\left(\rho+\bar{\theta}\right)+\frac{\lambda\bar{\theta}\bar{\psi}(1-\omega)}{(1-\alpha)}\frac{\alpha^{2}\Omega_{2}\left(H\right)}{\Omega_{1}\left(H\right)}L_{Y} \tag{A1.43}$$

Note that by the free-entry condition, we have $g_V = g_w = g_Y$. Using equations (A1.43) and (3.9), we reformulate the expected value of an innovation

$$\underbrace{\frac{1}{1-\alpha}\left(g_\phi + g_Z\right) + \left(\rho + \bar{\theta}\right) - \frac{\bar{\theta}\bar{\psi}(1-\omega)\alpha^2\Omega_2\left(H\right)}{\lambda\left(1-\alpha\right)\Omega_1\left(H\right)}\left(1-L_R\right) + \lambda L_R}_{=\mathrm{r}+\lambda L_R} - \underbrace{\frac{\lambda L_R}{1-\alpha}\left(\gamma_1+\gamma_2\right)}_{=\frac{\dot{V}(t)}{V(t)}}$$

$$= \underbrace{\frac{\alpha\gamma_1}{\lambda\left(1-\gamma_1\right)}\frac{\left(\chi+H\right)^{-\frac{\alpha}{1-\alpha}}}{\Omega\left(H\right)}\left(1-L_R\right)}_{=\frac{\pi\left(\bar{\phi}_{max},\underline{z}_{min}\right)}{V(t)}} \tag{A1.44}$$

From (A1.44), we find the equilibrium level of labor in R&D sector (see equation (3.27)).

3.J Proof of Proposition 1

To assess the impact catastrophe probability on labor in R&D, we take derivative of L_R (equation (3.27)) with respect to hazard rate $\bar{\theta}$;

$$\frac{\partial L_R}{\partial\bar{\theta}} = \frac{\left(\frac{\bar{\psi}\lambda(1-\omega)\alpha^2}{(1-\alpha)}\frac{\Omega_2(H)}{\Omega_1(H)} - 1\right)}{\left(\lambda + \frac{\lambda\alpha^2(1-\omega)\bar{\theta}\bar{\psi}}{1-\alpha}\frac{\Omega_2(H)}{\Omega_1(H)} + \frac{\alpha\gamma_1\lambda}{(1-\gamma_1)}\frac{(\chi+H)^{-\frac{\alpha}{1-\alpha}}}{\Omega(H)}\right)}$$

$$-\frac{\left(\frac{\bar{\psi}\lambda(1-\omega)\alpha^2}{(1-\alpha)}\frac{\Omega_2(H)}{\Omega_1(H)}\right)\left[\frac{\bar{\theta}\bar{\psi}\lambda(1-\omega)\alpha^2}{(1-\alpha)}\frac{\Omega_2(H)}{\Omega_1(H)} + \frac{\alpha\gamma_1\lambda}{(1-\gamma_1)}\frac{(\chi+H)^{-\frac{\alpha}{1-\alpha}}}{\Omega_1(H)} - \left(\rho+\bar{\theta}\right)\right]}{\left(\lambda + \frac{\lambda\alpha^2(1-\omega)\bar{\theta}\bar{\psi}}{1-\alpha}\frac{\Omega_2(H)}{\Omega_1(H)} + \frac{\alpha\gamma_1\lambda}{(1-\gamma_1)}\frac{(\chi+H)^{-\frac{\alpha}{1-\alpha}}}{\Omega(H)}\right)^2} \tag{A1.45}$$

The impact depends whether the penalty rate $\bar{\psi}$ is sufficiently high or not.

$$sign\left(\frac{\partial L_R}{\partial\bar{\theta}}\right) > 0$$

$$if\ \bar{\psi} > \frac{(\lambda+\rho)(1-\omega)+\sqrt{((\omega-1)(\lambda+\rho))^2+4\bar{\theta}\omega(\omega-1)\left(\lambda+\frac{\alpha\gamma_1\lambda}{(1-\gamma_1)}\frac{(\chi+H)^{-\frac{\alpha}{1-\alpha}}}{\Omega(H)}\right)}}{2\bar{\theta}\omega} \tag{A1.46}$$

$$sign\left(\frac{\partial L_R}{\partial \bar{\theta}}\right) < 0$$

$$if\ \bar{\psi} < \frac{(\lambda+\rho)(1-\omega)+\sqrt{((\omega-1)(\lambda+\rho))^2+4\bar{\theta}\omega(\omega-1)\left(\lambda+\frac{\alpha\gamma_1\lambda}{(1-\gamma_1)}\frac{(\chi+H)^{-\frac{\alpha}{1-\alpha}}}{\Omega(H)}\right)}}{2\bar{\theta}\omega} \tag{A1.47}$$

3.K Proof of Proposition 2

Taking the derivative of L_R (equation (3.27)) with respect to marginal cost of pollution H ;

$$\frac{\partial L_R}{\partial H} = \frac{[Z_1+Z_2]\left[\lambda+\rho+\bar{\theta}\right]}{\left(\lambda+\frac{\lambda\alpha^2(1-\omega)\bar{\theta}\bar{\psi}}{1-\alpha}\frac{\Omega_2(H)}{\Omega_1(H)}+\frac{\alpha\gamma_1\lambda}{(1-\gamma_1)}\frac{(\chi+H)^{-\frac{\alpha}{1-\alpha}}}{\Omega_1(H)}\right)^2} > 0 \tag{A1.48}$$

where

$$Z_1 = \frac{\lambda(1-\omega)\alpha^2\bar{\theta}\bar{\psi}}{(1-\alpha)}\left[\frac{\partial\Omega_2(H)}{\partial H}\frac{1}{\Omega_2(H)} - \frac{\Omega_2(H)}{(\Omega_1(H))^2}\frac{\partial\Omega_1(H)}{\partial H}\right]$$

$$Z_2 = \frac{\alpha\gamma_1\lambda}{(1-\gamma_1)}\left[-\frac{\partial\Omega_1(H)}{\partial H(\Omega_1(H))^2} - \frac{\alpha}{1-\alpha}\frac{(\chi+H)^{-\frac{\alpha}{1-\alpha}-1}}{\Omega_1(H)}\right]$$

The impact of pollution tax depends on the relationship between elasticity of aggregation factor of production $\Omega_1(H)$ and that of intermediate good demand. The increase of marginal cost of pollution increases labor allocation in R&D if

Condition 1.

$$-\frac{\frac{\partial\Omega_1(H)}{\Omega_1(H)}}{\frac{\partial H}{H}} > -\frac{\frac{\partial\Omega_2(H)}{\Omega_2(H)}}{\frac{\partial H}{H}} \tag{A1.49}$$

A necessary condition to have a positive impact of pollution tax on growth is that the elasticity of aggregation factor of production function is higher than the elasticity of aggregation factor of intermediate good factor. We know that a higher marginal pollution tax implies a lower production of final good which follows a lower intermediate good demand. Then, the term Z_1 is positive.

Condition 2.

$$H + \chi < 2 \tag{A1.50}$$

In order to ensure that Z_2 is positive, we impose some conditions on some key parameters of the model. We suppose that $\frac{\bar{\gamma}+1}{b} = 0$ and $\alpha = \frac{1}{3}$. Our purpose in doing this is to gain insight about the mechanism that explains why a higher marginal cost of pollution can boost the economic growth at the long run. If the producing cost of machines is sufficiently low and the Condition 1. is ensured, the nominator is positive. Consequently, the effect of pollution tax is positive on growth.

To assess the impact of hazard rate on the effect of environmental taxation, we compute

$$\frac{\partial}{\partial\bar{\theta}}\left(\frac{\partial L_R}{\partial H}\right) = \left(\frac{\left[k^2 - \left(\lambda+\rho+\bar{\theta}\right)\frac{\lambda\alpha^2(1-\omega)\bar{\psi}}{1-\alpha}\frac{\Omega_2(H)}{\Omega_1(H)}\right]\left[\frac{\lambda(1-\omega)\alpha^2\Upsilon_1}{(1-\alpha)} + \frac{\alpha\gamma_1\lambda\Upsilon_2}{(1-\gamma_1)}\right]}{k^4}\right)$$

$$-\left(\frac{k^2\left(\lambda+\rho+\bar{\theta}\right)\frac{\lambda\bar{\theta}\bar{\psi}(1-\omega)\alpha^2\Upsilon_2}{(1-\alpha)}}{k^4}\right) \tag{A1.51}$$

where $k = \lambda + \frac{\lambda\alpha^2(1-\omega)\bar{\theta}\bar{\psi}}{1-\alpha}\frac{\Omega_2(H)}{\Omega_1(H)} + \frac{\alpha\gamma_1\lambda}{(1-\gamma_1)}\frac{(\chi+H)^{-\frac{\alpha}{1-\alpha}}}{\Omega(H)}$ and $\Upsilon_1 = \frac{\partial\Omega_2(H)}{\partial H}\frac{1}{\Omega_2(H)} - \frac{\Omega_2(H)}{(\Omega_1(H))^2}\frac{\partial\Omega_1(H)}{\partial H} < 0$, $\Upsilon_2 = -\frac{\alpha(\chi+H)^{-\frac{\alpha}{1-\alpha}-1}}{(1-\alpha)(\chi+H)} - \frac{\partial\Omega_1(H)}{\partial H}\frac{1}{(\Omega_1(H))^2} \lesseqgtr 0.$

The derivative of (A1.48) yields a complicated term. However, one can remark that it is possible to write a third degree equation $f\left(\bar{\psi}\right)$ in order to find the roots for constant penalty rate $\bar{\psi}$. Since, we will have three different roots, we can analyze the implications of hazard rate on the effect of pollution tax H ;

$$sign\left(\frac{\partial}{\partial\bar{\theta}}\left(\frac{\partial L_R}{\partial H}\right)\right) > 0 \; if \; \bar{\psi} > g\left(.\right) \tag{A1.52}$$

$$sign\left(\frac{\partial}{\partial\bar{\theta}}\left(\frac{\partial L_R}{\partial H}\right)\right) < 0 \; if \; \bar{\psi} < g\left(.\right) \tag{A1.53}$$

where $g\left(.\right)$ is the positive root of the third degree equation $f\left(\bar{\psi}\right)$ which is a function of constant parameters of the model. We also verify this condition by a numerical analysis in the text.

3.L Proof of Proposition 5

The differentiation of (3.35) yields ;

$$\frac{dW^*}{dH} = \underbrace{\frac{\frac{\Omega_1(H)}{\alpha^2\Omega_2(H)Y(0)}\left[\frac{\partial\Omega_2(H)}{\partial H}\frac{1}{\Omega_2(H)} - \frac{\Omega_2(H)}{(\Omega_1(H))^2}\frac{\partial\Omega_1(H)}{\partial H} + \frac{\Omega_1(H)}{\alpha^2\Omega_2(H)Y(0)}\frac{dY(0)}{dH}\right]}{\rho+\bar{\theta}-g}}_{\text{Output effect}}$$

$$+\underbrace{\frac{log\left(\frac{\alpha^2\Omega_2(H)}{\Omega_1(H)}Y\left(0\right)\right) - \bar{\psi}\bar{\theta}\left(\omega + (1-\omega)\,log\left(\frac{\gamma_1(1-\alpha)\alpha^{\frac{2}{1-\alpha}}\phi_{max}(0)\underline{z}_{min}(0)}{1-\gamma_1}\right)\right)}{\left(\rho+\bar{\theta}-g\right)^2}\frac{dg}{dH}}_{\text{Growth effect}}$$

where the sign of $\frac{dY(0)}{dH}$ is negative since the green tax decreases the final good production.

On the other hand, since the labor in production shifts to the R&D sector, the growth rate of the economy increases. At the end, if the growth effect dominates the output effect, the green tax increases the welfare. It is easy to remark that when the necessary conditions for the positive effect of pollution tax on growth are not satisfied, both output and growth effect are negative. Consequently, the welfare becomes negative.

The effect of catastrophe probability on welfare is

$$\frac{dW^*}{d\bar{\theta}} = \underbrace{\frac{\frac{\Omega_1(H)}{\alpha^2\Omega_2(H)Y(0)}\frac{dY(0)}{d\bar{\theta}} - \bar{\psi}\left(\omega + (1-\omega)\,log\left(\frac{\gamma_1(1-\alpha)\alpha^{\frac{2}{1-\alpha}}\phi_{max}(0)\underline{z}_{min}(0)}{1-\gamma_1}\right)\right)}{\rho + \bar{\theta} - g}}_{\text{Output effect}}$$

$$-\underbrace{\frac{log\left(\frac{\alpha^2\Omega_2(H)}{\Omega_1(H)}Y\left(0\right)\right) - \bar{\psi}\bar{\theta}\left(\omega + (1-\omega)\,log\left(\frac{\gamma_1(1-\alpha)\alpha^{\frac{2}{1-\alpha}}\phi_{max}(0)\underline{z}_{min}(0)}{1-\gamma_1}\right)\right)}{\left(\rho + \bar{\theta} - g\right)^2}\left(1 - \frac{dg}{d\bar{\theta}}\right)}_{\text{Growth effect}}$$

Similar to the effect of green tax burden on welfare, the total effect of catastrophe probability on welfare depends on the growth and output effect. If the abrupt event probability pushes the market economy to invest more in R&D and the increase of the growth rate compensates the output effect, the welfare of the economy increases.

Chapter 4

Catastrophic Events, Sustainability and Limit Cycles

Can Askan Mavi

4.1 Introduction

The fact that uncertain catastrophic events could cause large scale damages is widely recognized (Alley et al. (2003), Field et al. (2012)). A large number of studies focus on decision making regarding the exploitation policy of natural resources under uncertainty (Bretschger and Vinogradova (2017), Tsur and Zemel (1998, 2007, 2016c), Clarke and Reed (1994)). In addition, some recent literature focuses on the optimal environmental policy to deal with the uncertainty. For this purpose, adaptation and mitigation policies and their implications under uncertainty are one of the major points of interest (Zemel (2015), Tsur and Zemel (2016a), Mavi (2017)).

Apart from studies on uncertainty and resource exploitation, another branch of the literature concentrates on the relationship between discounting and sustainability, which is one of the lasting important debate in the economics literature. Especially, the debate has been intensified in the context of climate change (Stern (2006), Weitzman (2007), Heal (2009)). Some of these studies is related to the role of individual time preferences (see Endress et al. (2014), Schneider et al. (2012), Marini and Scaramozzino (1995, 2008), Burton (1993)). The presence of individual time preferences in an economic model is appealing because infinitely lived agent model (ILA hereafter) is criticized for not respecting consumer's preferences[1].

The articles cited above which are incorporate the individual discount rate are based on the seminal framework proposed by Calvo and Obstfeld (1988). Authors introduce individual time preferences (i.e. individual discount rate) in an OLG model. Then, they find statically the aggregate consumption level of all generations at a given time. Once the aggregation is made, the model reduces to the representative agent framework. This framework has been used in environmental economics literature in order to study some important topics as inter-generational equity[2] by the above-cited papers. However, these papers do not analyze the role of individual time preferences on the aggregate long term dynamics. This clearly introduces a dichotomy between the OLG part and ILA model since one does not know the implications of individual discount rate on the long term dynamics in the ILA model . One of the aim of this chapter is to fulfill this need and to analyze the implications of the individual discount rate (or family discount rate[3]) on the aggregate dynamics in depth.

How to position this study in the literature? On one hand, studies treating the long term impacts of uncertainty on resource exploitation policy do not take into account the sustainability and intergenerational equity (see Bommier et al. (2015), Zemel (2015)). On the other hand, the strand of the literature on sustainability and intergenerational equity does not take into account the uncertain events (see Burton (1993); Endress et al. (2014); Marini and Scaramozzino (1995)). In this sense, to the best of our knowledge, we can argue that the link between the sustainability and catastrophic events is overlooked in environmental economics literature and this chapter aims to fill this gap.

[1]When there is only a unique social discount rate, we cannot distinguish individual impatience from social planner's impatience level. This is one of the ethical objections to ILA model.

[2]Contributions made in intergenerational equity discussions argue that ILA framework as a utilitarian social welfare function corresponds to a different generation at each point of time (see Schneider et al. (2012)).

[3]We use the notions family and individual discount rate interchangeably. We explain this choice in the remainder of the chapter.

In this chapter, we have two important motivations: first, we aim to show the importance of individual preferences for sustainability when the economy under catastrophic event uncertainty is exposed to limit cycles (Hopf bifurcation). Secondly, we show that limit cycles at the long run are optimal but not in conformity with the Sustainable Development criterion, which is one of the most prominent sustainability criterion. Then, we argue that the Sustainable Development criterion should be revised by policymakers to encompass the limit cycles as well. Otherwise, one should avoid these cycles from a normative point of view, even though they are optimal.

The contribution of this chapter is twofold: first, by extending the Calvo and Obstfeld (1988) framework to account for the uncertain events, we show that for some critical parameter values for the individual (family) discount rate, endogenous cycles (Hopf Bifurcation) arise in the economy at the long run. The mechanism behind limit cycles can be summarized as follows: on the one hand, the economy accumulates physical capital and creates waste. In this sense, the environment is used as a "sink" in the economy. This can be considered as an economic goal. On the other hand, since the inflicted damage after the catastrophic event is proportional to the remaining natural resource stock after the event, the economy wishes to protect the natural capital. This is the environmental goal. When it becomes difficult to steer between these conflicting policies, it may be optimal to cycle around the optimal steady state[4] (see Wirl (2004)).

What is the role of the catastrophic event probability on the limit cycles? Note that when there is no catastrophic event probability, the above-mentioned trade-off between the economic and environmental goal disappears since the environment has only a proactive role, meaning that the utility from the environment becomes effective once the catastrophic event occurs. Therefore, we show also that the individual preferences have no effect on the stability properties of the model.

In order to better understand the motivation of the chapter and the reason why we use an OLG model, here are some other clarifications regarding the link between individual preferences and the occurrence of the limit cycles: in fact, the existence of the limit cycles is possible even without an OLG model. One can easily show that the limit cycles take place in a representative agent model (see Wirl (2004)). In other words, the main source of

[4]Note that cycles around the steady state are optimal.

the bifurcations is the above-mentioned trade-off and not the structure of the population. However, this is not to say that individual discount rate does not matter for bifurcations. Individual discount rate is important in the sense that it can make more/less difficult to steer between the economic and environmental goal. For some levels of the individual discount rate, it becomes torn to decide between the environmental and economic goal. It is this difficulty, depending on the individual discount rate, that makes cycles appear. Therefore, we find important to focus on the individual discount rate in this study.

Indeed, since our aim in this chapter is to focus on the importance of individual preferences regarding the sustainability of the economy, we think it is convenient to use Calvo and Obstfeld (1988) framework which allows to distinguish the individual discount rate from the social planner's discount rate.

One may argue that the trade-off between the economic and environmental goal which is the source of the limit cycles is usual in growth models with environmental aspects. Alternatively, the occurrence of the limit cycles can also be understood through the complementarity of preferences (see Dockner and Feichtinger (1991), Heal and Ryder (1973))). For this purpose, we show that without the waste stemming from physical capital accumulation and the catastrophic event probability, the preferences of the economy over time are independent in the sense of Koopmans (see Koopmans (1960)). It follows that the economy admits a stable equilibrium at the long run.

How the complementarity of preferences over time can explain the limit cycles? For example, if there is an incremental increase of consumption near some date t_1, this implies a reallocation of the consumption between future dates t_2 and t_3 (for example some part of the consumption shifts from t_2 to t_3) if there is a complementarity of preferences over time, it follows that the consumption increases at date t_1 and t_3 and decreases at date t_2. If the complementarity of preferences are sufficiently strong, these fluctuations will occur in a loop forever.

Indeed, the limit cycles are studied extensively in environmental economics. Wirl (1999, 2004) and Bosi and Desmarchelier (2016, 2017) study the existence of the limit cycles in models with representative agent framework. Nonetheless, none of these studies link the limit cycles to equity across generations and sustainability in the sense of Sustainable

Development Criterion.

At this point, the question to be addressed is: what are the implications of the limit cycles regarding the sustainability? Note that the Sustainable Development Criterion requires that the utility of consumption should have a non-decreasing path (i.e $\frac{du(c(t))}{dt} \geq 0$). If the economy is exposed to the limit cycles due to the trade-off between the environmental and the economic goal and/or due to the complementarity of preferences, the Sustainable Development Criterion is not respected since the utility and the dynamics of natural resource stock have cyclical behavior at the long run.

Secondly, contrary to the Calvo and Obstfeld (1988) framework and to the articles using this framework, we show that individual time preferences can change the stability properties of the model. This result disproves the conventional result which states that aggregate dynamics are solely governed by the social planner's discount rate (see Endress et al. (2014), Schneider et al. (2012), Marini and Scaramozzino (1995, 2008), Burton (1993)). Indeed, we show that the individual discount rate has an important role regarding the aggregate long term dynamics, hence the sustainability of the economy.

Since the first part of the model is an OLG model, there is an intra-generational allocation of consumption which is stable over time. We also show that intra-generational equity can be conform with the sustainability since we show that a more equal allocation of consumption between generations ensures a stable equilibrium at the long run.

As stated above, there are two options: either a policy maker should revise the Sustainable Development criterion to encompass the limit cycles, or one should avoid these cycles to be in conformity with this criterion. One can argue that the second option is better, since the sustainability and intergenerational equity are generally perceived as normative inquiries (Solow (2005, 2006)). Then, a social planner who pays attention to the sustainability and intergenerational equity should seek to avoid the limit cycles. We show that the social planner can avoid the limit cycles by an environmental policy aiming at the protection of the environment. This is possible due to the fact that a higher natural resource stock implies a lower marginal utility of consumption. As a result, different levels of the individual discount rate are expected to not change much the trade-off between the economic and the environmental goal. Consequently, there is less chance that an economy is trapped in limit cycles in the long run.

The remainder of the chapter is organized as follows. The section 4.2 presents the benchmark model and explains all economic mechanisms behind the limit cycles in detail. Section 4.3 explains the model with environmental protection that can avoid limit cycle and the last section 4.4 concludes the chapter.

4.2 Model

We use the Calvo and Obstfeld (1988) framework to answer the questions that we motivated in the introduction. Our model is a mix of Weil (1989) and Calvo and Obstfeld (1988). Weil (1989) proposes a model with infinitely-lived families where the population grows. In this study, the economy consists of many infinitely-lived families whose size decreases non-stochastically over time at a constant pace, so that the population remains constant at each time t. Indeed, the members within the family die with some probability but there is no uncertainty about the size of the family that vanishes when the time t tends to infinity. The reason to make this assumption of constant population, differently from Weil (1989), is as follows: since Calvo and Obstfeld (1988) discount the lifetime utilities of representative agents of each cohort to be born and representative agents currently alive, it is necessary to have a constant population (or families with a decreasing size over time). Otherwise, from the date 0, there would be an infinite number of people already alive at date 0[5]. In our framework, at each instant of time t, there is a newly-born family of size 1. As of time t, the size of the family is $e^{-h(t-s)}$. Then, at each instant of time t, there is $\int_0^\infty e^{-h\tau} d\tau$ *unity* of family. In our framework, it is more appropriate to use the term "unit" than "number" for families. At each time t, there is always a constant $\int_0^\infty e^{-h\tau} d\tau$ unit of family at the aggregate level. However, there is an infinite number of family of unit between 0 and 1[6].

Differently from Calvo and Obstfeld (1988), *our social planner discounts the utility of families and not the utility of a representative individual facing a death probability.* The reason behind this reasoning is the following: it is difficult to deal with the death probability of

[5]Since the generation born at -∞ also grows, there would be an infinite number of individual

[6]For example, it is correct to say that there is 0.005 unit of family of some age τ.

individuals when the horizon of the maximization problem of a representative individual is limited to the catastrophic event date (let this date T). Since the age of death is a random variable between $[0; \infty]$[7], an individual can die before or after the catastrophic event. Therefore, one should split the expected utility of a representative individual[8] in two parts; the first part considers the case where the agent dies before T and the second part, the case where the agent dies after T. Unfortunately, this yields some tedious calculations and makes impossible to have closed-form solutions. However, when the social planner considers the utility of a given family, it is unambiguous that the family does not vanish even though there are people who die within the family (before or after the catastrophic event).

Considering the overall utility of a family instead of the utility of a representative individual of the family (cohort) allows to maintain the social planner's objective simple since we do not deal with the death probability in the objective function of the social planner.

Note also that once the social planner allocates the consumption to families of different ages at each time, she does not care about the allocation of the consumption within the family. We implicitly assume that each member of the family is identical.

4.2.1 The economy's structure

A family born at time b lives infinitely with a subjective discount rate $\beta > 0$ which is the individual discount rate of family members[9]. A family of age $\tau = t - b$ maximizes the utility until the unknown catastrophic event date T. For the sake of simplicity, we assume a single-occurrence catastrophic event as it is widely used in the literature (see Bommier et al. (2015); Clarke and Reed (1994); Polasky et al. (2011)). Examples of single occurrence event imply naturally arising pathogens, organisms, like the biotechnology products, can devastate ecosystem functions. Another illustrative examples can be crop failures, human overpopulation and non-sustainable agriculture. After the occurrence of the single-occurrence doomsday event, we assume that the consumption reduces to a constant level of consumption c_{min} (see Tsur and Zemel (2016c)). The subsistence level of

[7]The age of death can also be bounded by an arbitrary value (see Bommier and Lee (2003)).
[8]The expectancy term is for the death probability.
[9]All the members of the family have β as a discount factor.

consumption is supposed not provide any utility to individuals (i.e $u(c_{min}) = 0$). After the catastrophic event, the economy is exposed to a catastrophic damage which is proportional to the resource stock level S at the time of the catastrophic event T. The consequences of the catastrophic event are described by the post event value $\varphi(S)$ that we discuss in the remainder of the chapter.

A family born at the birth date b enjoys $c(b,t)$ until the catastrophic event date T. The size of the family declines non-stochastically and constantly at rate h . The family exists even after the catastrophic event T but the consumption is reduced to c_{min}. The family (*and not the representative individual of the family*) maximizes the utility

$$V_b = \int_b^T u(c(b,t))\, e^{-(\beta+h)(t-b)}dt + \int_T^\infty \underbrace{u(c_{min})\, e^{-(\beta+h)(t-b)}dt}_{=0} \tag{4.1}$$

It is important to note that h is the constant death probability for individuals within the family (see Blanchard (1984)) but at the family scale, it holds for the decrease rate of the size for a family. In our framework, the term $e^{-h(t-b)}$ plays a similar role as in the models where the population growth (the decrease of the population in our case) is taken into account in the utility function[10]. Nonetheless, in our case, the population within the family decreases rather than it increases, then the family's discount rate is augmented by the rate of decrease of the size. In other terms, $c(b,t)$ can be understood as the consumption per capita of the family born at time b.

4.2.2 The planner's program

The social planner's objective is composed of two components. The first integral holds for the lifetime utilities of the families to be born, as measured from the birth date b until the catastrophic event date T. The second is an integral of utilities of the families currently alive. It is assumed that the social planner discounts families at rate $\rho > 0$. After the

[10]In a classical Ramsey growth model, the social planner maximizes $\int_0^\infty u(C(t))\, e^{-\rho t}dt$ where $C(t)$ is the aggregate consumption level. Assume that the population grows at rate n, if we want to express the maximization program by variables per capita, we can simply write $\int_0^\infty L(t)\, u(c(t))\, e^{-\rho t}dt = \int_0^\infty u(c(t))\, e^{-(\rho-n)t}dt$.

catastrophic event, the state of the economy is described by the post-event value function $\varphi(S)$ that depends on the natural resource stock and the economy is exposed to an inflicted damage which is proportional to the level of the natural stock. The welfare at time $t=0$ is

$$W(0)=\int_{-\infty}^{0}\left\{\int_{0}^{T}u(c(b,t))\,e^{-(\beta+h)(t-b)}dt\right\}e^{-\rho b}db+\int_{0}^{T}\left\{\int_{b}^{T}u(c(b,t))\,e^{-(\beta+h)(t-b)}dt\right\}e^{-\rho b}db$$
$$+e^{-\rho T}\varphi(S(T)) \tag{4.2}$$

All generations currently alive and to be born in the future are treated in a symmetric fashion to ensure the time-consistency of the problem. It follows that all families are discounted back to their birth date rather than to the beginning of the planning horizon. After changing the order of the integration by expressing the utility of generations in terms of the age τ (see Appendix (4.A) for details), we have

$$W(0)=\int_{0}^{T}\left\{\int_{0}^{\infty}u(c(b,t))\,e^{-(\beta+h-\rho)\tau}d\tau\right\}e^{-\rho t}dt+e^{-\rho T}\varphi(S(T)) \tag{4.3}$$

The flow of utility on any date before T is the integral over all families of instantaneous utilities discounted by their "family" discount rate and the social planner's discount rate. The integral of these utility flows until T with the post-event value gives $W(0)$. The maximization problem of the social planner is solved in two stage. In the first stage, the social planner allocates the consumption between families of different ages for a given $C(t)$. In the second stage, the social planner chooses the aggregate consumption path.

It is important to know what happens after the catastrophic event. Following Tsur and Zemel (2016b), we write the post-event value with the corresponding damage function $\psi(S)$

$$\varphi(S)=(u(c_{min})-\psi(S)) \tag{4.4}$$

where the damage function $\psi(S)$ is described as

$$\psi(S) = \bar{\psi}(\omega_1 - \omega_2 logS) \tag{4.5}$$

where ω_1 is the unrecoverable part of the penalty. The parameter ω_1 is supposed to be high such that some part of the damage is irreversible. The use of the penalty function implies that the more the natural capital stock is protected, the less the economy suffers from the penalty[11].

4.2.3 A static problem : the utility of families

Before finding the social optimum, one should find first the optimal allocation of consumption between families (or generations). Then, after aggregating the consumption of each family, we can take the expectations of the social planner's program in order to take into account the distribution of T since the occurrence date of the catastrophic event is uncertain.

Define aggregate consumption

$$C(t) = \int_0^\infty c(t-\tau, t)\, e^{-h\tau} d\tau \tag{4.6}$$

Note that the consumption of a family is proportional to its size. Therefore, it is necessary to take the decrease of the size into account. Given a level of the aggregate consumption $C(t)$ at time t, the social planner should allocate the consumption across families. The indirect utility is

$$U(C(t)) = \max_{\{c(t-\tau,t)\}_{\tau=0}^{\infty}} \int_0^\infty u(c(t-\tau,t))\, e^{-(\beta+h)\tau} d\tau \tag{4.7}$$

subject to $\int_0^\infty c(t-\tau,t)\, e^{-h\tau} d\tau \leq C(t)$ (see Appendix (4.C) for the static optimization program). The following graphic illustrates the idea of the optimal distribution of the consumption across families;

[11] Tsur and Zemel (1998) use a similar specification for the penalty function. However, authors have a pollution stock instead of the natural capital stock. In this case, the penalty rate increases with respect to the pollution stock after the catastrophic event.

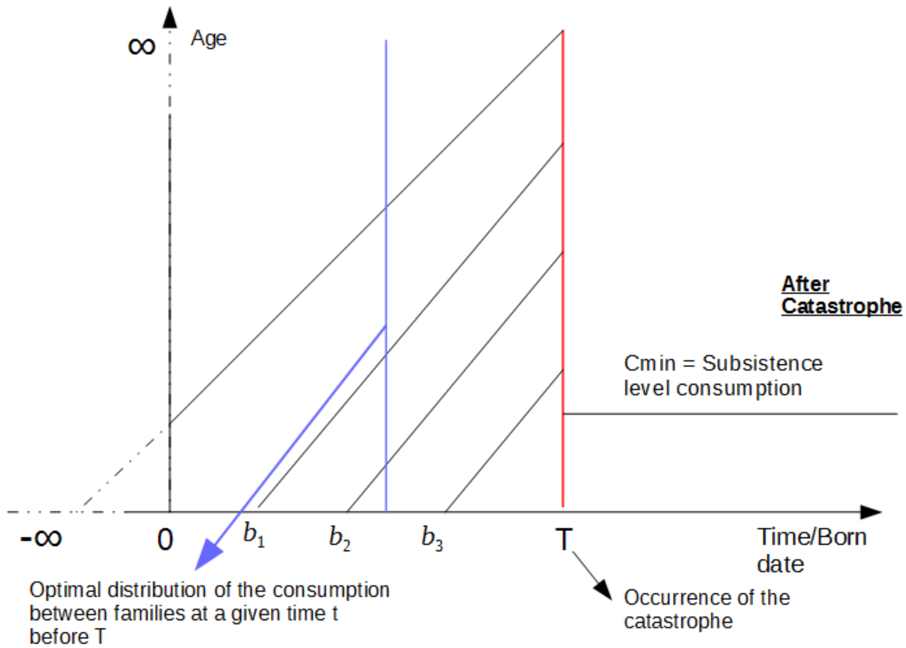


Figure 4.1: Allocation of the consumption across families

After the aggregation of the consumption (see Appendix (4.C)), the social planner's program (4.3) becomes

$$W(0) = \int_0^T U(C(t))\, e^{-\rho t} dt + e^{-\rho T} \varphi(S(T)) \tag{4.8}$$

4.2.4 A dynamic problem : social optimum

To solve the maximization problem by standard optimal control techniques, one should find the deterministic equivalent of the objective function (4.8) which is a stochastic expression since the catastrophic event date T is unknown (see Kamien and Schwartz (1978)). Therefore, we take the expectations of (4.8) and find

$$\max_{C(t)} W(0) = \int_0^\infty \{U(C(t)) + \theta\varphi(S(t))\}\, e^{-(\rho+\theta)t} dt \tag{4.9}$$

where θ is the exogenous catastrophic event probability. The economy is subject to two constraints that are for physical capital accumulation K and for the natural resource stock S which regenerates with respect to a logistic growth function. Note that the natural capital accumulation is negatively affected by the physical capital accumulation which is similar to Wirl (2004) and Ayong Le Kama (2001). In this sense, the nature is considered as a "sink" for the waste coming from the physical capital. In fact, this feature creates a trade-off between capital accumulation and we will analyze in greater depth its negative effects on the nature in this study. The social planner maximizes (4.9) subject to

$$\begin{cases} \dot{K}(t) = f(K(t)) - \delta K(t) - \int_0^\infty c((t-\tau,t))\, e^{-h\tau} d\tau \\ \dot{S}(t) = G(S(t)) - \gamma f(K(t)) \end{cases} \tag{4.10}$$

Once the social planner allocates the consumption across families, we need to find the utility of consumption at the aggregate level. The use of CRRA form utility function gives (see Appendix (4.C) for details.)

$$U(C(t)) = \frac{Z_1^\sigma C(t)^{1-\sigma} - Z_2 c_{min}^{1-\sigma}}{1-\sigma} \tag{4.11}$$

where $Z_1 = \left(\frac{\sigma}{(\beta+\sigma h-\rho)}\right)$ and $Z_2 = \frac{1}{\beta+h-\rho}$ are the aggregation terms, which include individual discount rate. The regeneration of the environment and the physical capital accumulation are described as

$$G(S) = (1-S)\, S \tag{4.12}$$

$$f(K) = AK \tag{4.13}$$

The social planner maximizes the program (4.9) subject to physical and natural capital accumulation constraints (4.10). The current-value Hamiltonian for maximizing W

$$\mathcal{H} = U(C) + \theta\varphi(S) + \lambda(f(K) - \delta K - c) + \mu(G(S) - \gamma f(K)) \tag{4.14}$$

The first order conditions and dynamics of the economy are as follows

$$\begin{cases} U_C = Z_1^{\sigma} C^{-\sigma} = \lambda \\ \dot{K} = f(K) - \delta K - Z_1 \lambda^{-\frac{1}{\sigma}} \\ \dot{S} = G(S) - \gamma f(K) \\ \dot{\lambda} = (\rho + \theta)\lambda - (f_K - \delta)\lambda + \mu\gamma f_K \\ \dot{\mu} = (\rho + \theta)\mu - \mu G_S - \theta \varphi_S \end{cases} \tag{4.15}$$

By using the functional forms above, the steady-state of the economy can be written as a function of the renewable resource stock S :

$$\begin{cases} K^*(S) = \frac{(1-S^*)S^*}{\gamma A} \\ \lambda^*(S) = \frac{\gamma A \mu^*(S)}{((A-\delta)-(\rho+\theta))} \\ \mu^*(S) = \frac{\bar{\psi}\omega_2\theta}{S^*((\rho+\theta)-(1-2S^*))} \end{cases} \tag{4.16}$$

Proposition 1. *In an economy with the catastrophic event probability, a Hopf bifurcation occurs at two different values for the critical bifurcation parameter β .*

Proof. See Appendix (4.D)

It is important to figure out the economic reasons behind the occurrence of the limit cycles. On one hand, the more the natural capital stock is conserved, the less the penalty rate is (see Tsur and Zemel (1998)). It follows that the disutility coming from the penalty decreases with respect to the natural resource stock. Then, the environment plays an important role when the catastrophic event takes place. This creates an incentive to protect the environment which can be referred to as an *environmental goal*. On the other hand, the resource stock in this model is used as a sink for the waste coming from physical capital accumulation[12]. This represents the *economic goal*. Trying to decide between these two opposite strategies can lead to the limit cycles around the steady state when it becomes torn to steer between two conflicting goals (see Wirl (2004)).

[12]See Ayong Le Kama (2001) and Wirl (2004) for a similar model.

One may argue that this trade-off growth models with the environment does not cause necessarily instability. As stated before, another possible source of the limit cycles can be *the complementarity of preferences over time*[13] (see Dockner and Feichtinger (1991)). To understand the complementarity over time, suppose that an incremental increase of consumption at time t_1 occurs. If this incremental increase shifts the preferences of consumption from t_3 to t_2 or vice versa [14], there is complementarity of preferences over time. If an incremental change of consumption at time t_1 does not shift the preferences of other dates, this means that preferences are intertemporally independent (see Koopmans (1960)).

The model is shown to have complementarity of preferences over time when there is waste coming from physical capital accumulation (see Appendix (4.G)). To understand the implications of complementarity regarding the occurrence of limit cycles, consider an increase of consumption near date t_1 which shifts a part of the consumption of date t_3 to t_2. At time t_1, the physical capital accumulation decreases. It follows that the economy accumulates less waste due to a lower physical capital accumulation and this leads to a higher stock of natural capital. On the contrary, since at date t_3, consumption decreases and the economy accumulates more physical capital. Then, there is more waste accumulation that harms the environment and so on. From this mechanism, we can understand the intuition behind the limit cycles. Due to the complementarity over time, natural resource stock increases and decreases regularly at consecutive dates.

Note that this mechanism does not hold if there is no catastrophic event probability or waste stemming from the physical capital accumulation. We show that when the waste rate γ or the catastrophic event probability θ is equal to zero, the complementarity over time vanishes. (see Appendix (4.G).) When there is no waste in the economy, we can also remark that the dynamic system (4.15) reduces to a block recursive system of (K, λ) and (S, μ) which admits a saddle path equilibrium. It is worthwhile noting that the limit cycles occur when synergistic between control and state and/or between states is strong (Wirl (1992)). In the model, it is evident that waste accumulation creates a strong link between two state variables.

The legitimate question to be addressed is why and how individual preferences cause the

[13] Dockner and Feichtinger (1991) and Heal and Ryder (1973) show that the limit cycles and unstable behavior can stem from interdependent preferences over time.

[14] i.e.. adjacent complementarity and distant complementarity vice versa.

limit cycles and unstable behavior in this economy. It is evident that the individual discount rate β is crucial to explain cyclical behavior in the economy since it changes the difficulty of steering between the economic and environmental goal. This can be remarked through the utility of aggregate consumption which depends on the individual discount factor.

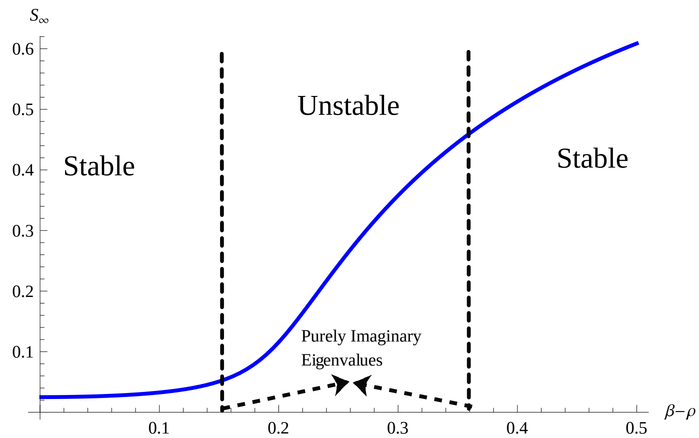


Figure 4.2: Steady state levels of natural resource stock with respect to the difference between individual discount rate β and the discount rate of social planner ρ

Despite the fact that we try simple functional forms, it is always a hard task to find analytically the critical value for the bifurcation parameter. (see Wirl (1999, 2004))) As in Wirl (2004), we refer to a numerical analysis to study in which cases bifurcation is a possible outcome[15]. Figure (4.2) shows the steady state values of natural resource stock with respect to individual discount rate β.

When $\beta = 0.973625$ and $\beta = 1.167842$, we have a pair of purely imaginary eigenvalues where Hopf bifurcation occurs. The limit cycles are shown to be stable at low and high levels of individual discount rate β[16]. When we have β between 0.973625 and 1.167842, the

[15]For the numerical exercise in the rest of the analysis, we use the relaxation algorithm proposed by Trimborn et al. (2008). The method consists of determining the solution by an initial guess and improves the initial guess by iteration. Since the iteration improves the solution progressively, it relaxes to the correct solution.

[16]For $\beta = 0.925644$ and $\beta = 0.974355$, the Lyapunov numbers are -0.011645276 and -0.0116452956 respectively. This means that model shows a sub-critical Hopf bifurcation at two different critical individual discount rate β.

economy is exposed to unstable spirals. As mentioned above, it is possible to understand why the limit cycles and unstable spirals take place but we are unable to offer an exact economic explanation why Hopf bifurcation takes place at given two different critical values [17].

According to the Figure (4.2), it is clear that the economy becomes more conservative to exploit natural resources when the individual discount rate becomes higher than the social planner's discount rate. This can be explained by the fact that the social planner shifts the consumption to younger families that have longer lifespan. A higher individual discount rate causes a decrease in the marginal utility of consumption and also a decrease in the shadow price of the natural resource stock, which is followed by a lower exploitation of natural resources at the long run (see the steady-state of the economy 4.16).

The dynamic system is stable when the difference between individual and social planner discount rate is close to zero and also when the difference is sufficiently high. However, when there is only a modest difference between the individual and social planner's discount rate, the strategy is torn between the economic objective and the environmental goal and steering between these two conflicting policies cause the limit cycles.

Definition 1. *A path of utility respects the Sustainable Development criterion if* $dU\left(C\left(t\right)\right)/dt \geq 0$.

The Sustainable Development criterion states that the utility of consumption should follow a non-decreasing or at least a constant path in order to ensure the sustainability of the economy. This means that limit cycles for consumption violate sustainability criterion since consumption decreases at some moments of time t. Then, Sustainable Development criterion is violated when we have $\beta \in [0.973625.., 1.167842..]$.

An important point is that the model reduces to a representative agent model at the second stage. Consequently, we cannot talk about the intergenerational equity as in Schumacher and Zou (2008) for the aggregate dynamics because we do not know the length of the life cycle of a generation along time t. For example, each generation can see its consumption decrease or increase in the same way. Then, the equity between generations can be respected.

[17]See Wirl (1992, 1994) for a detailed discussion regarding the difficulty of giving intuitive economic explanations for bifurcation points.

For this reason, we focus on the link between the limit cycles and the sustainability in our analysis.

Proposition 2. *When the social planner treats all generations equally (i.e. $\beta = \rho$), sustainability criterion is respected*[18].

According to the Figure (4.2), another interesting result is that the way how a social planner allocates the consumption over different generations has an impact on the long term dynamics of the economy (i.e. sustainability). Treating generations equally ensures also the sustainability at the long run since the economy admits a stable equilibrium. This shows a complementarity between intra-generational equity and sustainability. However, when the unequal consumption allocation across families (i.e $\beta \neq \rho$) is moderate (two vertical dashed lines and the area between in the Figure 4.2), it is likely that the economy traps in the limit cycles which compromise the sustainability. In addition, when the social planner distributes the consumption across families in a very unequal manner, the economy admits stability.

Figure (4.3) shows the limit cycles for given parameters in a phase diagram with plane (K, S)[19]

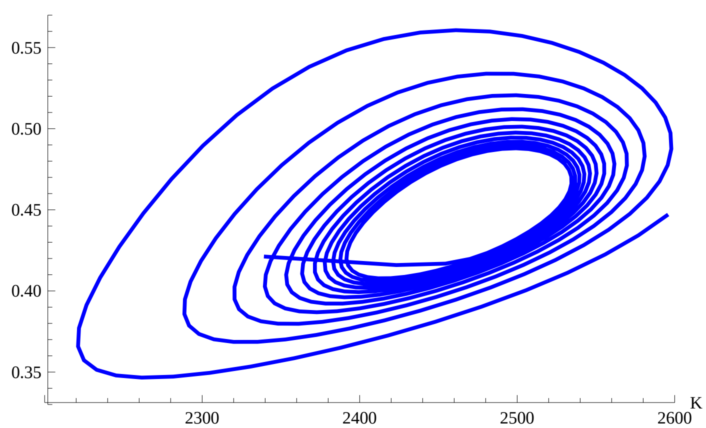


Figure 4.3: Limit cycles on a phase plane (K, S) with bifurcation parameter β

[18]Note that we cannot put exactly $\beta = \rho$ since the aggregation term Z_1 and Z_2 turn out to be indeterminate. In the numerical analysis, we skip the point where the individual discount is exactly equal to the social planner's discount rate.

[19]The phase diagram and dynamics of aggregate consumption are plotted for the second critical bifurcation point where $\beta = 1.167842$.

A closer intuitive explanation to the mechanism explained for the limit cycles in this study is proposed by Heal (1982) and Bosi and Desmarchelier (2016) which are based on *compensation effect*. Assume that the economy is at the steady state at a given date t and assume that the natural capital stock S [20] decreases exogenously. The degradation of the natural capital pushes agents to increase their consumption since they would like to compensate the disutility due to the decrease of natural capital stock. It follows that the capital accumulation decreases as well. Then, the natural capital increases as there is less waste and so on: deterministic cycles arise. Since in our specification, the objective function is separable in consumption and natural resource stock, we are unable to justify the limit cycles by a compensation effect. However, we show that even with a non-additive objective function, we show that the limit cycles exist.

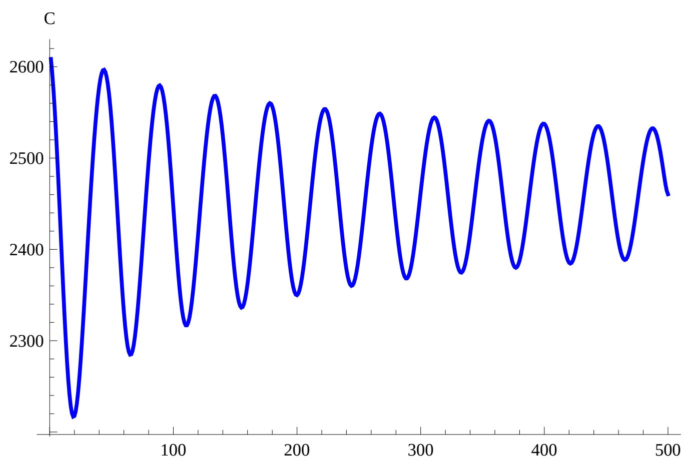


Figure 4.4: Limit cycles across time for consumption

The model stipulates that not only the level of natural resource stock or a catastrophic event as a fact would violate the Sustainable Development criterion but even individual preferences can compromise sustainability. Figure (4.4) shows that at the first bifurcation point, the utility is exposed to cycles and does not converge to a stable point. From a normative point of view, a social planner that pays attention to sustainability should avoid any path that leads to bifurcations. We figure out the policy possibilities to avoid limit cycles in the following section.

[20]Note that Bosi and Desmarchelier (2016) use pollution stock instead of natural capital stock in their model.

The role of the catastrophic event probability

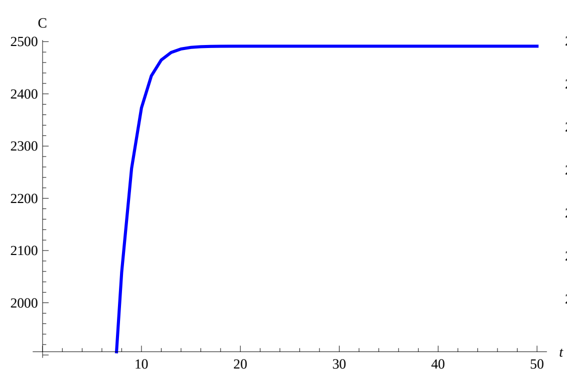
What is the role of the catastrophic event probability regarding the limit cycles? We analyze an economy without uncertainty in order to show that the catastrophic event probability has important implications on the aggregate dynamics at the long run. For this purpose, we set the catastrophic event probability to zero. This version of the model is shown to always admit a saddle-path stable equilibrium at the long run (see Proposition 1 in the Appendix (4.E)).

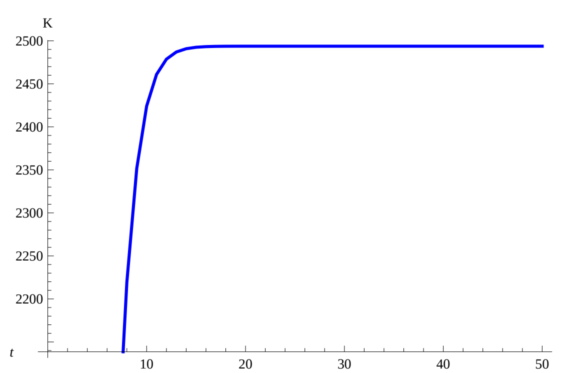
Proposition 3. *(a) In an economy without a catastrophic event, the individual discount rate β has no effect on long term dynamics. (b) The economy admits a saddle path equilibrium when there is not a catastrophic event probability.*

Proof. *See Appendix (4.E).*

This result is quite plausible, since the trade-off betwwen the positive effect of the natural resource stock on the damage rate (environmental goal) and the use of the environment as a sink for the waste (economic goal) is absent when there is no catastrophic event probability.

The aggregate dynamics in the economy without catastrophic event probability are illustrated by the following graphics;





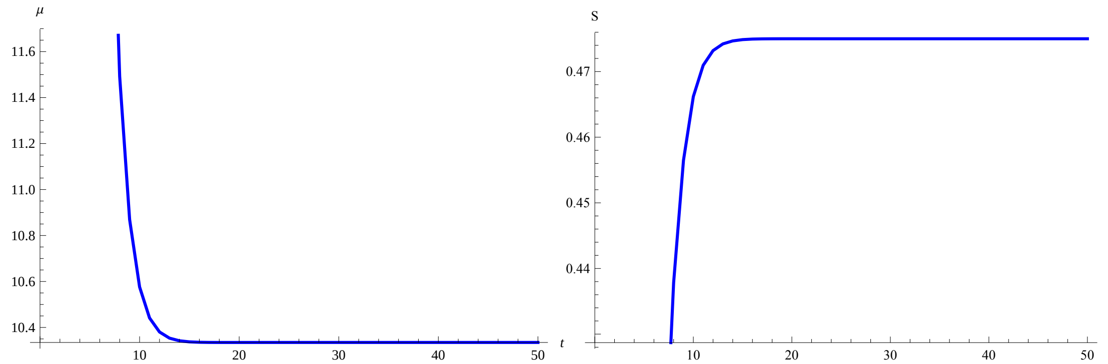


Figure 4.5: Aggregate dynamics without the catastrophic event probability

As expected, when there is no catastrophic event probability, the economy has a monotonic behavior and converges to a stable steady state at the long run. In this version of the model, an important point is that social planner does not face a trade-off between the environmental and economic goal. She takes into account only the negative effect of the capital accumulation on the environment. Then, the environment is just served as a sink and does not represent any amenity value. To sum up, the economy does not have any cyclical behavior on the transition to steady or at the long run.

4.3 Model with abatement activities

In this section, we focus on the implications of abatement activities on the limit cycles. The social planner's program with abatement activities is

$$W = \int_0^{\infty} \left\{ U\left(C\left(t\right)\right) + \theta\varphi\left(S\left(t\right)\right) \right\} e^{-(\rho+\theta)t} dt \tag{4.17}$$

The dynamics of the capital accumulation contain the cost of mitigation M.

$$\begin{cases} \dot{K}\left(t\right) = f\left(K\left(t\right)\right) - \delta K\left(t\right) - C\left(t\right) - M\left(t\right) \\ \dot{S}\left(t\right) = G\left(S\left(t\right)\right) + \Gamma\left(M\left(t\right)\right) - \gamma f\left(K\left(t\right)\right) \end{cases} \tag{4.18}$$

where

$$\Gamma(M) = M^{\alpha}\,,\ \alpha > 0$$

holds for the abatement activities such as reforestation, desalination of water stock, enhancing carbon sinks, etc. The specification of the abatement is along the same lines with Chimeli and Braden (2005). Alternatively, the function $\Gamma(M)$ can be considered as "an environmental protection function". The cost for environmental protection may be directed not only toward pollution mitigation but also toward the protection of forests and the recovery of degraded areas. Equivalently, the abatement activity in this model helps improve the environmental quality. Another possibility would be to reduce the waste by an environmental policy. However, this would lead to more complicated dynamics since the control variable M would depend directly on the physical capital K.

The first order conditions and dynamics of the economy with the abatement activity are as follows

$$\begin{cases} U_C = Z_1^{\sigma}\lambda^{-\frac{1}{\sigma}} = \lambda \\ \dot{K} = f(K) - \delta K - Z_1\lambda^{-\frac{1}{\sigma}} - M \\ \dot{S} = G(S) + \Gamma(M) - \gamma f(K) \\ \dot{\lambda} = (\rho+\theta)\lambda - (f_K - \delta)\lambda + \mu\gamma f_K \\ \dot{\mu} = (\rho+\theta)\mu - \mu G_S - \theta\varphi_S \end{cases} \tag{4.19}$$

The steady-state of the economy as a function of the natural resource stock is

$$\begin{cases} K^*(S) = \frac{(1-S^*)S^* + \left(\alpha\frac{\mu^*(S^*)}{\lambda^*(S^*)}\right)^{\frac{\alpha}{1-\alpha}}}{\gamma A} \\ M^*(S) = \left(\alpha\frac{\mu(S^*)}{\lambda(S^*)}\right)^{\frac{1}{1-\alpha}} \\ \lambda^*(S) = \frac{\gamma A\mu^*(S^*)}{((A-\delta)-(\rho+\theta))} \\ \mu^*(S) = \frac{\bar{\psi}\omega_2\theta}{S^*((\rho+\theta)-(1-2S^*))} \end{cases} \tag{4.20}$$

Proposition 4. *(i) The Abatement activity makes the limit cycles less likely to occur by increasing the determinant and decreasing the sum of sub-matrices of Jacobian. Then, the* *economy admits a saddle-path stable equilibrium.*

(ii) When $M^* = 64.1321$*, the economy admits a stable equilibrium.*

Proof. See Appendix (4.F).

We also show numerically that economy with abatement activities always admits a saddle path equilibrium.

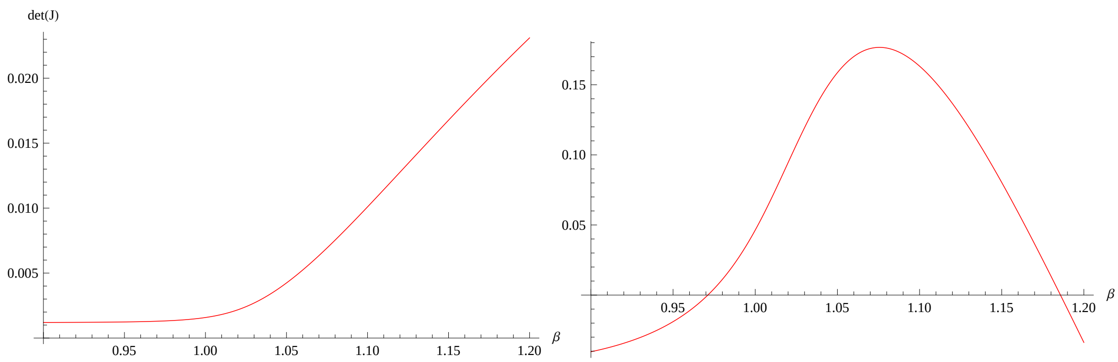


Figure 4.1: $det\,(J)$ and Ω in benchmark model

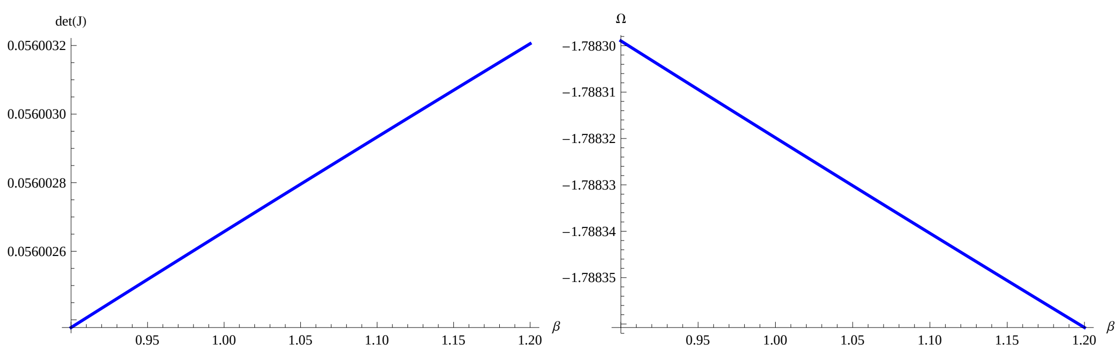


Figure 4.2: $det\,(J)$ and Ω in the augmented model

It is obvious that the determinant and the sum of sub matrices of the model with the abatement activity increase and decrease respectively in the model with the abatement activity. Numerical analysis confirms that whatever the level of individual discount rate is, the economy always admits a saddle path equilibrium. The economic explanation is the following: when the steady state level of the natural resource stock is higher due to the environmental protection activity, the marginal utility of consumption (see (4.20)) is lower. In this case, the variations of consumption with respect to different levels of individual discount rate β would have a lower impact on marginal utility. Indeed, when the economy is trapped in the limit cycles due to a tight trade-off between the economic and environmental goal, the abatement activity relaxes this tight trade-off by lowering the marginal utility of consumption. Then, the economy escapes the dilemma between the environmental and economic goal by giving more weight to the environmental goal.

4.4 Conclusion

In this chapter, we showed the existence of the limit cycles in an economy exposed to the catastrophic event probability. The limit cycles are caused by conflicting economic and environmental goals and/or by the complementarity of preferences over time. An interesting finding of the chapter is that individual time preferences of agents other than the social planner's discount rate is crucial not only for intra-generational equity but also for the sustainability of an economy. It is shown that when the individual discount rate is close to social planner's discount rate, intra-generational equity and sustainability criterion at the long run are respected. This result also disproves a widespread result in the literature which says that aggregate dynamics are solely governed by the social planner's discount rate (see Endress et al. (2014); Schneider et al. (2012)). From a normative point of view, the limit cycles are considered as an undesirable result since it compromises the prominent Sustainable Development Criterion. Therefore, either a policymaker should revise the Sustainable Development criterion to encompass the limit cycles or avoid them. The model with the environmental protection activities shows that it is less likely to have the limit cycles at the long run with an environmental policy aiming at improving/protecting the environment.

Appendix

4.A Change of the order of integration

We use the theorem of Fubini-Tonelli to change the order of the integration. We write

$$W(0) = \int_{-\infty}^{0} \left\{ \int_{0}^{T} u\left(c\left(b,t\right)\right) e^{-(\beta+h)(t-b)} \mathbb{1}_{t \geq b} dt \right\} e^{-\rho b} db + \int_{0}^{T} \left\{ \int_{0}^{T} u\left(c\left(b,t\right)\right) e^{-(\beta+h)(t-b)} \mathbb{1}_{t \geq b} dt \right\} e^{-\rho b} db$$

$$+ e^{-\rho T} \varphi\left(S\left(T\right)\right) \tag{C.1}$$

By using the Chasles relation, it is possible to write down the problem in the following form

$$\int_{-\infty}^{T} \left\{ \int_{0}^{T} u\left(c\left(b,t\right)\right) e^{-(\beta+h)(t-b)} \mathbb{1}_{t \geq b} dt \right\} e^{-\rho b} db + e^{-\rho T} \varphi\left(S\left(T\right)\right)$$

Since the lower and upper bounds of integrals do not depend on the variables of integration, it is easy to change the order of integration

$$\int_{0}^{T} \left\{ \int_{-\infty}^{T} u\left(c\left(b,t\right)\right) e^{-(\beta+h)(t-b)} e^{\rho(t-b)} \mathbb{1}_{t \geq b} db \right\} e^{-\rho t} dt + e^{-\rho T} \varphi\left(S\left(T\right)\right) \tag{C.2}$$

In order to get rid of the indicator function, by knowing that $t < T$ (when $t > T$, each family has a minimum consumption level) and $b \leq t$, we can write

$$\int_{0}^{T} \left\{ \int_{-\infty}^{t} u\left(c\left(b,t\right)\right) e^{-(\beta+h)(t-b)} e^{\rho(t-b)} db \right\} e^{-\rho t} dt + e^{-\rho T} \varphi\left(S\left(T\right)\right) \tag{C.3}$$

Since we know $\tau = t - b$, we can switch from the generational index b to age index τ

$$\int_0^T \left\{ \int_0^\infty u\left(c\left(b,t\right)\right) e^{-(\beta+h-\rho)\tau} d\tau \right\} e^{-\rho t} dt + e^{-\rho T} \varphi\left(S\left(T\right)\right)$$

4.B Aggregate Economy facing a catastrophic event

Taking the expectations of (4.8) gives

$$E_T\left[\int_0^T U\left(C\left(t\right)\right) e^{-\rho t} dt + e^{-\rho T} \varphi\left(S\left(T\right)\right)\right] \tag{C.4}$$

Note that probability distribution and density function are

$$f\left(t\right) = \theta e^{-\theta t} \ \ and \ \ F\left(t\right) = 1 - e^{-\theta t} \tag{C.5}$$

We write the following expression

$$\int_0^\infty f\left(T\right)\left[\int_0^T U\left(C\left(t\right)\right) e^{-\rho t} dt + e^{-\rho T} \varphi\left(S\left(T\right)\right)\right] dT \tag{C.6}$$

$$= \underbrace{\int_0^\infty f\left(T\right)\left[\int_0^T U\left(C\left(t\right)\right) e^{-\rho t} dt\right] dT}_{\text{A}} + \underbrace{\int_0^\infty f\left(T\right)\left[e^{-\rho T} \varphi\left(S\left(T\right)\right)\right] dT}_{\text{B}} \tag{C.7}$$

Integrating by parts A

$$dX = f\left(T\right) \implies X = \int_0^T f\left(s\right) ds$$

$$Y = \int_0^T U\left(c\left(t\right)\right) e^{-\rho t} \implies dY = U\left(c\left(T\right)\right) e^{-\rho T}$$

Using $\int Y dX = XY - \int X dY$ yields

$$A = \left[\left(\int_0^T f(s)\, ds \right) \left(\int_0^T U(c(t))\, e^{-\rho t} dt \right) \right]_{T=0}^{\infty} - \int_0^{\infty} F(T)\, U(c(T))\, e^{-\rho T} dT \qquad \text{(C.8)}$$

Recall that $\int_0^\infty f(s)\, ds = 1$. The part A leads to

$$\int_0^\infty U(c(t))\, e^{-\rho t} dt - \int_0^\infty F(t)\, U(c(t))\, e^{-\rho t} dt \qquad \text{(C.9)}$$

Taking the overall sum $A + B$, we have

$$\int_0^\infty \left[(1 - F(t))\, U(c(t)) + f(t)\, \varphi(S(t)) \right] e^{-\rho t} dt \qquad \text{(C.10)}$$

Inserting the probability distribution and the density function gives

$$\int_0^\infty \left[U(c(t)) + \theta \varphi(S(t)) \right] e^{-(\rho+\theta)t} dt \qquad \text{(C.11)}$$

4.C Utility of each generation (family)

In order to find the utility of each family, we write down the Lagrangian in the following way

$$\mathcal{L} = \int_0^\infty u(c(t-\tau, t))\, e^{-(\beta+h-\rho)\tau} d\tau + \lambda(t) \left[C(t) - \int_0^\infty c((t-\tau, t))\, e^{-h\tau} d\tau \right] \qquad \text{(C.12)}$$

From the Lagrangian, we find the utility of each family

$$u_c(c(t-\tau, t))\, e^{-(\beta-\rho)\tau} = u_c(c(t,t)) \qquad \text{(C.13)}$$

where $u_c(c(t,t))$ is the marginal utility of consumption for a newly-born family. Using a utility function of form CRRA $\frac{c(t-\tau,t)^{1-\sigma} - c_{min}^{1-\sigma}}{1-\sigma}$, we have

$$c\left(t-\tau,t\right)=c\left(t,t\right)e^{-\frac{\left(\beta-\rho\right)\tau}{\sigma}} \tag{C.14}$$

Aggregating the consumption of all generations gives

$$C\left(t\right)=\int_{0}^{\infty}c\left(t-\tau,t\right)e^{-h\tau}d\tau=\int_{0}^{\infty}c\left(t,t\right)e^{-\frac{\left(\beta-\rho\right)\tau}{\sigma}}e^{-h\tau}d\tau=Z_{1}c\left(t,t\right) \tag{C.15}$$

where$Z_1=\left(\frac{\sigma}{\left(\beta+\sigma h-\rho\right)}\right)$. Plugging the equation (C.14) in the maximization program (4.7) yields

$$U\left(C\left(t\right)\right)=\max_{\left\{c\left(t-\tau,t\right)\right\}_{\tau=0}^{\infty}}\int_{0}^{\infty}\frac{\left[c\left(t,t\right)e^{-\frac{\left(\beta-\rho\right)\tau}{\sigma}}\right]^{1-\sigma}-c_{min}^{1-\sigma}}{1-\sigma}e^{-\left(\beta+h-\rho\right)\tau}d\tau \tag{C.16}$$

Substituting the equation (C.15) in (C.16) gives

$$U\left(C\left(t\right)\right)=\left[\frac{C\left(t\right)}{Z_{1}}\right]^{1-\sigma}\int_{0}^{\infty}\frac{\left[e^{-\frac{\left(\beta-\rho\right)\tau}{\sigma}}\right]^{1-\sigma}-c_{min}^{1-\sigma}}{1-\sigma}e^{-\left(\beta+h-\rho\right)\tau}d\tau \tag{C.17}$$

After some simple algebra, we have

$$U\left(C\left(t\right)\right)=\frac{Z_{1}^{\sigma}C\left(t\right)^{1-\sigma}-Z_{2}c_{min}^{1-\sigma}}{1-\sigma} \tag{C.18}$$

where $Z_2=\frac{1}{\beta+h-\rho}$.

4.D Proof of Proposition 1

The Jacobian matrix of the differential system with using given functional forms is

$$J = \begin{bmatrix} A-\delta & 0 & \frac{Z_1}{\sigma}\lambda^{-\frac{1}{\sigma}-1} & 0 \\ -A\gamma & (1-2S) & 0 & 0 \\ 0 & 0 & (\rho+\theta)-(A-\delta) & A\gamma \\ 0 & 2\mu+\frac{\theta\bar{\psi}\omega_2}{S^2} & 0 & (\rho+\theta)-(1-2S) \end{bmatrix} \tag{C.19}$$

Following Dockner and Feichtinger (1991), the characteristic polynomial associated with Jacobian is

$$\upsilon^4 - trJ\upsilon^3 + b_2\upsilon^2 - b_3\upsilon + det\,(J) = 0 \tag{C.20}$$

where b_2 and b_3 are the sum of second and third order minors of Jacobian respectively. We have

$$trJ = 2\,(\rho+\theta) \quad and \quad -b_3 + (\rho+\theta)\,b_2 - (\rho+\theta)^3 = 0 \tag{C.21}$$

The eigenvalues of the Jacobian matrix are calculated from first order conditions.

$$\upsilon_i = \frac{(\rho+\theta)}{2} \pm \sqrt{\left(\frac{\rho+\theta}{2}\right)^2 - \frac{\varOmega}{2} \pm \sqrt{\varOmega^2 - det\,(J)}} \tag{C.22}$$

The sum of the determinants of sub-matrices of Jacobian can be specified

$$\varOmega = \begin{bmatrix} A-\delta & \frac{Z_1}{\sigma}\lambda^{-\frac{1}{\sigma}-1} \\ 0 & (\rho+\theta)-(A-\delta) \end{bmatrix} + \begin{bmatrix} (1-2S) & 0 \\ 2\mu+\frac{\theta\bar{\psi}\omega_2}{S^2} & (\rho+\theta)-(1-2S) \end{bmatrix} + 2\begin{bmatrix} 0 & 0 \\ 0 & A\gamma \end{bmatrix} \tag{C.23}$$

Then we have

$$\varOmega = (A-\delta)\left[(\rho+\theta)-(A-\delta)\right] + (1-2S)\left[(\rho+\theta)-(1-2S)\right] \tag{C.24}$$

and

$$det\,(J) = [(\rho+\theta) - (1-2S)]\,[(A-\delta)\,(1-2S)\,[(\rho+\theta) - (A-\delta)]] + (A\gamma)^2\,\frac{Z_1\lambda^{-\frac{1}{\sigma}-1}}{\sigma}\left(2\mu + \frac{\theta\bar{\psi}\omega_2}{S^2}\right) \tag{C.25}$$

For the possibility of a Hopf bifurcation, Ω should be positive. In this framework, this one is possible when the following condition is ensured

$$\rho + \theta > G_S > 0 \tag{C.26}$$

In this case, we can observe that an economy in which all steady-stae levels of the natural resource stock exceeding maximum sustainable yield is stable. A Hopf bifurcation in a $4x4$ dimension system occurs when two of the eigenvalues have only imaginary parts. This means that the real part of these two eigenvalues crosses zero for some parameters. More precisely, the derivative of the real part of eigenvalues with respect to the critical bifurcation parameter is non-zero. The necessary condition to have a Hopf bifurcation

$$det\,(J) - \left(\frac{\Omega}{2}\right)^2 > 0 \tag{C.27}$$

$$det\,(J) - \left(\frac{\Omega}{2}\right)^2 - (\rho+\theta)^2\,\frac{\Omega}{2} = 0 \tag{C.28}$$

are necessary and sufficient in order to have four complex eigenvalues and two having only imaginary parts. A saddle-path equilibrium is a possible outcome if and only if

$$det\,(J) > 0 \;\; and \;\; \Omega < 0 \tag{C.29}$$

In addition, the inequality $0 < det\,(J) < \frac{1}{2}\Omega^2$ is sufficient in order to have all the eigenvalues with real parts, which implies local monotonicity. Unfortunately, it is not possible to show analytically where the Hopf bifurcation occurs for the critical parameter β. Therefore, we make a numerical analysis and show that the conditions (C.27) and (C.28) hold for two different values of the individual discount rate β. When $\beta = 0.973625$ and $\beta = 1.167842$, Hopf bifurcations occur. The following two graphics shows numerically the critical parameter values for the individual discount rate β where Hopf bifurcations occur.

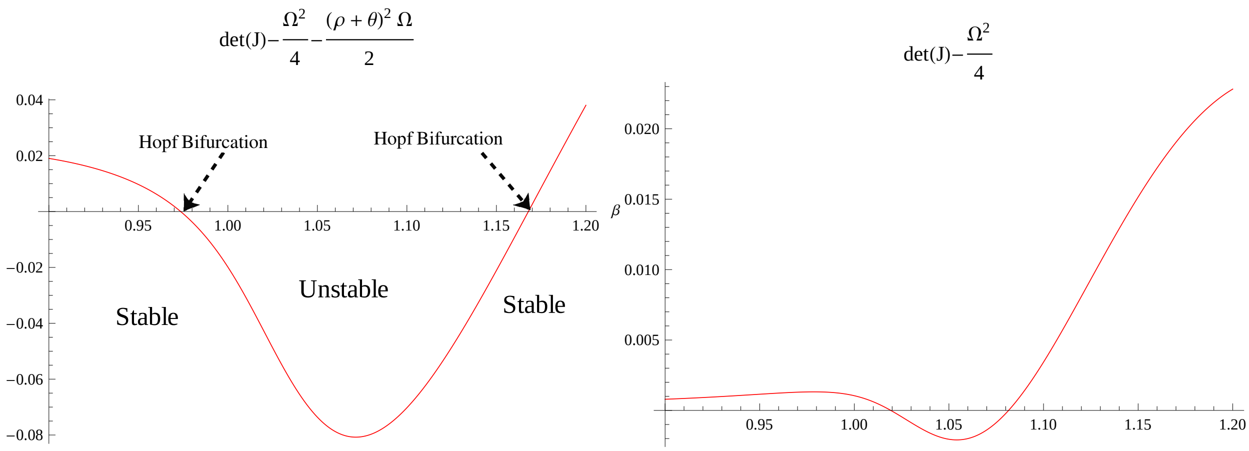


Figure 4.D.1: Conditions for Hopf Bifurcation

4.E Proof of Proposition 3

The maximization program is indeed the reduced form of the social welfare function.

$$W^{NC} = \int_0^{\infty} U\left(C\left(t\right)\right) e^{-\rho t} dt \tag{C.30}$$

subject to

$$\begin{cases} \dot{K}\left(t\right) = f\left(K\left(t\right)\right) - \delta K\left(t\right) - C\left(t\right) \\ \dot{S} = G\left(S\right) - \gamma f\left(K\left(t\right)\right) \end{cases} \tag{C.31}$$

The differential system describing dynamics of the economy is

$$\begin{cases} \dot{K} = f\left(K\right) - \delta K - Z_1 \lambda^{-\frac{1}{\sigma}} \\ \dot{S} = G\left(S\right) - \gamma f\left(K\right) \\ \dot{\lambda} = \rho\lambda - \left(f_K - \delta\right)\lambda + \mu\gamma f_K \\ \dot{\mu} = \rho\mu - \mu G_S \end{cases} \tag{C.32}$$

In an economy without catastrophic events, some straightforward calculations allow us to find analytically the steady state equilibrium.

$$\begin{cases} S^* = \frac{1-\rho}{2} \\ K^*(S) = \frac{(1-S^*)S^*}{\gamma A} \\ \lambda^*(S) = \left(\frac{(A-\delta)K^*(S)}{Z_1}\right)^{-\sigma} \\ \mu^*(S) = \frac{\lambda^*(S)[(A-\delta)-\rho]}{\gamma A} \end{cases} \tag{C.33}$$

Then, just recall the $det\,(J)$ becomes the following one when there is not a catastrophic event

$$det\,(J) = 2\,(A\gamma)^2 \left(\frac{(A-\delta)}{(A-\delta)-\rho}\left(\frac{1-\rho}{2}\right)\left(1-\frac{1-\rho}{2}\right)\right) > 0 \tag{C.34}$$

$$\Omega = (A-\delta)\,(\rho-(A-\delta)) < 0 \tag{C.35}$$

Then, it is easy to remark that the aggregation term Z_1 cancels out when there is not a catastrophic event probability. It is also obvious that Ω does not depend on Z_1 neither.

4.F Proof of Proposition 4

The Jacobian matrix of the differential system with abatement activity at steady state

$$J = \begin{bmatrix} A-\delta & 0 & \frac{Z_1}{\sigma}\lambda^{-\frac{1}{\sigma}-1} + \frac{1}{1-\alpha}\left(\alpha\frac{\mu}{\lambda}\right)^{\frac{1}{1-\alpha}}\lambda^{-1} & -\frac{1}{1-\alpha}\left(\alpha\frac{\mu}{\lambda}\right)^{\frac{1}{1-\alpha}}\mu^{-1} \\ -A\gamma & (1-2S) & 0 & 0 \\ 0 & 0 & -\frac{\alpha}{1-\alpha}\left(\alpha\frac{\mu}{\lambda}\right)^{\frac{\alpha}{1-\alpha}}\lambda^{-1} & \frac{\alpha}{1-\alpha}\left(\alpha\frac{\mu}{\lambda}\right)^{\frac{\alpha}{1-\alpha}}\mu^{-1} \\ 0 & 2\mu+\frac{\theta\bar{\psi}\omega_2}{S^2} & 0 & (\rho+\theta)-(1-2S) \end{bmatrix} \tag{C.36}$$

The sum of the determinants of sub-matrices and determinant of Jacobian are

$$\Omega = (A-\delta)\left[(\rho+\theta)-(A-\delta)\right]+(1-2S)\left[(\rho+\theta)-(1-2S)\right]-\left(2\mu+\frac{\theta\bar{\psi}\omega_2}{S^2}\right)\frac{\alpha M^{\alpha}}{(1-\alpha)\mu} \tag{C.37}$$

and

$$det\,(J) = \left[(\rho+\theta)-(1-2S)\right]\left[(A-\delta)(1-2S)\left[(\rho+\theta)-(A-\delta)\right]\right]$$
$$+(A\gamma)^2\frac{Z_1\lambda^{-\frac{1}{\sigma}-1}}{\sigma}\left(2\mu+\frac{\theta\bar{\psi}\omega_2}{S^2}\right) \tag{C.38}$$

The proof is easy to follow. Let $S^A > S^B$ where S^A is the steady state level of natural stock with abatement activity and S^B stands for the steady state level of natural stock for the benchmark model without the abatement activity. By replacing the steady state value of μ in (C.37), we can reformulate the sum of sub-matrices of Jacobian

$$\Omega = (A-\delta)\left[(\rho+\theta)-(A-\delta)\right]+\left(1-2S^A\right)\left[(\rho+\theta)-\left(1-2S^A\right)\right]-\underbrace{\frac{\alpha M^{\alpha}}{(1-\alpha)}\left(\frac{1-(\rho+\theta)}{S^A}\right)}_{>0} \tag{C.39}$$

Notice that M depends only on constant parameters in the long run. The presence of abatement activity decreases the sum of sub-matrices of Jacobian Ω, which makes less likely that unstable spirals and cycles occur. To see the effect of a higher natural stock level on the determinant of the Jacobian matrix, we look at the first derivative with respect to S

$$\frac{\partial\,(det\,(J))}{\partial S} = \underbrace{((\rho+\theta)-(A-\delta))}_{<0}2\,(A-\delta)\left[2\left(1-2S^A\right)-(\rho+\theta)\right] \tag{C.40}$$

$$\underbrace{-\frac{(A\gamma)^2}{\sigma}Z_1\left(2\frac{\mu}{\lambda}+\frac{\theta\bar{\psi}\omega_2}{S^2\lambda}\right)\frac{\partial\lambda\left(S^A\right)}{\partial S}}_{<0}+\underbrace{\frac{((A-\delta)-(\rho+\theta))\,(1-(\rho+\theta))}{\gamma A S^2}}_{>0}>0$$

The two last terms are unambiguously negative and positive respectively. The determinant increases unambiguously with respect to S if $2\left(1-2S^{A}\right)-(\rho+\theta)<0$.

4.G Complementarity between different time periods

This proof aims to support the idea that the complementarity of preferences over time is an explanatory element for the existence of the limit cycles. The objective of the proof is also to point out the fact that complementarity over time vanishes when the waste rate coming from the physical capital accumulation γ is equal to zero.

We write down the objective function in the following form

$$J\left[c\left(.\right)\right]=\int_{0}^{\infty}v\left(C\left(t\right),S\left(t\right)\right)e^{-\left(\rho+\theta\right)t}dt \tag{C.41}$$

where $v\left(C\left(t\right),S\left(t\right)\right)=U\left(C\left(t\right)\right)+\theta\varphi\left(S\left(t\right)\right)$

In order to simplify the calculations, we do not take the quadratic form for regeneration function $G\left(S\right)$ but simply assume a linear regeneration function $G\left(S\right)=mS$.[21] At this point, Wirl (1992, 1994) show that the occurrence of limit cycles does not have any link with the form of regeneration function. The author shows that a linear regeneration function can also generate limit cycles. Our proof also supports this possibility by showing the existence of the complementarity of preferences over time. Then, the use of a linear regeneration function does not enter into a conflict with the aim of the proof. We can express physical and natural capital from constraints

$$S\left(t\right)=e^{mt}\gamma A\int_{t}^{\infty}K\left(s\right)e^{-ms}ds \tag{C.42}$$

$$K\left(t\right)=e^{\left(A-\delta\right)t}\int_{t}^{\infty}C\left(s\right)e^{-\left(A-\delta\right)s}ds \tag{C.43}$$

[21] Otherwise, in case of the use of logistic growth function for natural regeneration, one should deal with Riccati differential equation which yields very tedious calculations.

To understand the complementarity effects between state variables and control and state variables, we refer to Volterra derivatives (Heal and Ryder (1973), Dockner and Feichtinger (1991)). This requires to look at the marginal rate of substitution between different time periods t_1,t_2, t_3 etc. For example, the marginal utility at time t_1 is $J^{'}[C(.), t_1]$ which is a Volterra derivative. To sum up, a small incremental increase of consumption in the neighborhood of time t_1 can be calculated by using Volterra derivatives. The concept of Volterra derivative is useful to show how a change in consumption at a given date shifts the allocation of consumption between other dates. The marginal rate of substitution between consumption at dates t_1 and t_2

$$R[C(.), t_1, t_2] = \frac{J^{'}[C(.), t_1]}{J^{'}[C(.), t_2]} \tag{C.44}$$

In order to see the effect of an incremental change of consumption near date t_3, we take the Volterra derivative of $R[C(.), t_1, t_2]$.

$$R^{'}[C(.), t_1, t_2; t_3] = \frac{J^{'}[C(.), t_2]\, J^{''}[C(.), t_1, t_3] - J^{'}[C(.), t_1]\, J^{''}[C(.), t_2, t_3]}{\left(J^{'}[C(.), t_2]\right)^2} \tag{C.45}$$

If $R^{'}[C(.), t_1, t_2; t_3] > 0$, an incremental increase of consumption at date t_3 shifts consumption from t_2 to t_1. Therefore, there is complementarity between t_1 and t_3. This represents a *distant* complementarity. If $R^{'}[C(.), t_1, t_2; t_3] < 0$, the preferences shift from t_1 to $t_2$where two neighboring dates hold for *adjacent* complementarity. Taking the derivatives of (C.41) , (C.42) and (C.43),

$$J^{'}[C(.), t_1] = e^{-(\rho+\theta)t_1} v_C(C(t_1), S(t_1)) + f_C(C(t_1))\, e^{-mt_1} \int_{t_1}^{\infty} e^{-(\rho+\theta-m)t} v_s(C(t), s(t))\, dt \tag{C.46}$$

$$J^{''}\left[C\left(.\right),t_1,t_2\right]=f_C\left(C\left(t_1\right)\right)f_C\left(C\left(t_2\right)\right)e^{-m(t_1+t_2)}\int_{t_2}^{\infty}e^{-(\rho+\theta-2m)t}v_{ss}\left(C\left(t\right),s\left(t\right)\right)dt \tag{C.47}$$

where f_C is the derivative of S with respect to C. For the equation (C.42), we know that S is a function of C since K depends on C . For example, when the consumption varies marginally, the capital accumulation changes and consequently the trajectory of the natural capital accumulation changes as well. Hence, we can simply say $S=f\left(C\right)$ but we cannot know the form of this function analytically out of steady state (see Dockner and Feichtinger (1991)). Note also that since we have an additive objective function, u_{ss} does not depend directly on c in our model.

For the sake of simplicity regarding the analysis, we can restrict our attention to a constant investment path (i.e. steady state) similar to Heal and Ryder (1973) and Dockner and Feichtinger (1991). Using equations (C.42) and (C.43), we can write $S^*=\frac{\gamma AC^*}{m(A-\delta)}$. It is obvious that with this simplification, we can find the form of $f_C=\frac{\gamma A}{m(A-\delta)}>0$. We write simplified form of equations (C.46) and (C.47),

$$J^{'}\left[C\left(.\right),t_1\right]=e^{-(\rho+\theta)t_1}\left[v_C+\frac{f_Cv_s}{(\rho+\theta-m)}\right] \tag{C.48}$$

$$J^{''}\left[C\left(.\right),t_1,t_2\right]=\left(f_C\right)^2e^{-m(t_1+t_2)-(\rho+\theta-2m)t_2}\frac{v_{ss}}{\rho+\theta-2m} \tag{C.49}$$

With all these elements, it is easy to express the effect of marginal increase of consumption at date t_3 on the marginal rate of substitution between t_1 and t_2,

$$R^{'}\left[C\left(.\right),t_1,t_2;t_3\right]=\frac{\left(f_C\right)^2\frac{v_{ss}}{\rho+\theta-2m}}{v_C+\frac{f_Cv_s}{(\rho+\theta-m)}}e^{(\rho+\theta)(t_2-t_1)}\left[\alpha\left(t_3-t_1\right)-\alpha\left(t_3-t_2\right)\right] \tag{C.50}$$

where $0<t_1<t_2$. and $v_{ss}<0$. We know that the date t_2 and t_3 is placed after t_1 but we do not know the order of dates for t_2 and t_3. In order to understand the effect of a small increase near date t_3, we claim that the date t_3 is situated before t_2. Otherwise, it is evident in the expression above that there is no effect of a variation of the consumption at the date t_3 on the marginal rate of substitution between dates $t_1$and t_2. This makes sense because when we are near date t_3, all decisions at t_1 and t_2 are already made. Similar to Heal and Ryder (1973), we write

$$\alpha\left(t\right)=e^{-\left(\rho+\theta-m\right)t}\,for\,t>0 \tag{C.51}$$

$$\alpha\left(t\right)=e^{mt}\,for\,t<0 \tag{C.52}$$

We observe that $R^{'}<0$ which means that there exists an adjacent complementarity between dates t_2 and t_3.

$$t_3<\frac{\left(\rho+\theta-m\right)t_1-mt_2}{\left(\rho+\theta-2m\right)} \tag{C.53}$$

Note that right hand side of the inequality increases when θ increases. The limit cycles can appear in the model with both adjacent and distant complementarity. This is in line with the results of Dockner and Feichtinger (1991). However, we do not focus on the type of complementarity which is out of the scope of this study.

It is important to point out the presence of the waste in the model. For this purpose, we write the dynamics of the model when there is no waste coming from the physical capital accumulation.

$$\begin{cases}\dot{K}=f\left(K\right)-\delta K-Z_1\lambda^{-\frac{1}{\sigma}}\\ \dot{S}=G\left(S\right)\\ \dot{\lambda}=\left(\rho+\theta\right)\lambda-\left(f_K-\delta\right)\lambda\\ \dot{\mu}=\left(\rho+\theta\right)\mu-\mu G_S-\theta\varphi_S\end{cases} \tag{C.54}$$

We remark that the dynamical system without the waste reduces to a block-recursive system. This means that the state and co-state variables (K,λ) and (S,μ) evolve independently. From this feature in the model, one can understand that the dynamics of consumption c and the natural resource stock S are independent, meaning that $R^{'}\left[C\left(.\right),t_1,t_2;t_3\right]=0$

since $f_C = 0$ when there is no waste coming from the physical capital accumulation in the economy. Then, the complementarity over time vanishes.

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Résumé

Cette thèse est consacrée à l'étude des implications de l'incertitude sur la politique environnementale. Le débat autour de l'incertitude s'est intensifié dans le contexte du changement climatique et de durabilité. De nombreuses études récentes examinant les politiques environnementales ont montré comment l'incertitude peut modifier les comportements économiques. La thèse contribue à cette littérature croissante sur l'incertitude et la politique environnementale.

A cet effet, le chapitre 2 vise à présenter une nouvelle explication pour les trappes à pauvreté par la présence de la probabilité de catastrophe. Je présente un nouvel arbitrage entre les politiques d'adaptation et d'atténuation autres que l'arbitrage dynamique habituel mis en évidence dans de nombreuses études.

De nombreux rapports récents des institutions internationales ont commencé à mettre en évidence l'importance de construire une économie de marché grâce à des innovations en R&D qui gèrent les investissements d'adaptation et d'atténuation. Le chapitre 3 construit un modèle de croissance schumpétérienne dans lequel les investisseurs gèrent les investissements d'adaptation et d'atténuation.

Le chapitre 4 porte sur les préférences individuelles et la durabilité. Ce chapitre vise à montrer que le critère du développement durable peut ne pas être conforme aux décisions optimales dans un modèle économique avec une possibilité de catastrophe lorsqu'il existe des cycles limites (Hopf bifucation). Par conséquent, le critère devrait être révisé par les décideurs politiques pour inclure la possibilité des cycles limites.

Mots-clés : Catastrophes, Incertitude, Politique Environnementale, Durabilité, Préférences Individuelles

Summary

This thesis is dedicated to studying the implications of uncertainty on the design of the environmental policy. The debate around the uncertainty has intensified in the context of the climate change and sustainability. Many recent studies focusing on the environmental policies started to show how the uncertainty component can change the optimal behavior in the economy. The thesis contributes to this recent growing literature of uncertainty and environmental policy.

Chapter 2 aims to present a new explanation for poverty traps, by the presence of catastrophe probability. I present a new trade-off between adaptation and mitigation policies other than the usual dynamic trade-off highlighted in many studies.

Many recent policy reports of international institutions started to highlight the importance of building a market economy through R&D innovations that handles adaptation and mitigation investments. Chapter 3 builds a Schumpeterian growth model in which investors handle the adaptation and mitigation investments. I also show the implications of a catastrophic event risk on investment decisions. The results suggest that the economy can increase investments in R&D even though there is a higher risk of a catastrophic event.

Chapter 4 focuses on the individual preferences and sustainability. This chapter aims to show that the Sustainable Development criterion can be not in conformity with the optimal decisions in an economic model when there are limit cycles (Hopf bifurcation). Therefore, the criterion should be revised by policymakers to encompass the possibility of limit cycles.

Keywords : Catastrophes, Uncertainty, Environmental Policy, Sustainability, Individual Preferences